# IMPERATIVES DENOTE ACTIONS*

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## 1 Introduction

Following Szabolcsi 1982, Naumann 2001, and Lascarides & Asher 2003, this paper promotes actions as an essential element in semantic analysis. It proposes that imperatives denote actions, and speculates about embedding a variant of Segerberg's Dynamic Logic for imperatives within a Linear Logic treatment of imperatives and deontics along the lines of Barker 2010. The key test case will be Ross' Paradox and its deontic analog, the problem of free choice permission.

So what are actions? Actions change the world. This means that actions can be characterized by before-and-after pictures, that is, by a picture of the world before the action is performed, and a picture of the world afterwards. Technically, then, an action will be a relation over worlds, a set whose elements are ordered pairs $\langle w, w' \rangle$ where $w$ is the world before the action and $w'$ is the world after the action in question has been performed.

In previous work (Barker 2010), I argue that free choice permission requires a resource-sensitive logic like Linear Logic (see, e.g, Restall 2000, Oehrle 2003). The action-based account here is developed using a fragment of Dynamic Logic (e.g., Segerberg 1990), which is also resource-sensitive in the relevant sense (as explained below). The Linear Logic approach has some advantages over Dynamic Logic, including providing a unified account of disjunction, and a general account of negation. However, the Dynamic Logic account is still well worth considering, if only for the simplicity and clarity with which it addresses Ross' Paradox. Furthermore, I will suggest that Dynamic Logic is a faithful approximation of the Linear Logic account of Barker 2010, in the sense that there is a translation (along lines suggested by Koji Mineshima, personal communication) of Dynamic Logic into Linear Logic that respects the key inferences involved in Ross' Paradox and the problem of free choice permission. The hope is that the simplicity and the semantic clarity of Dynamic Logic can illuminate the mysterious denotational semantics of the Linear Logic account.

*Version of January 31, 2012. Thanks to Ana Aguilar, Koji Mineshima, Michael Moortgat, Floris Roelofsen, James Pustejovsky, Will Starr, and audiences at Columbia, Michigan State, NYU, Wayne State, and Sinn und Bedeutung 16.

## 2 Motivating actions in linguistic semantics

The three main clause types that are best represented cross-linguistically (Portner 2005) are declaratives, interrogatives, and imperatives. Declaratives are overwhelmingly the best studied. Declaratives typically express a proposition, and for our simple purposes here, we can assume that a proposition corresponds to a set of worlds. For instance, *John left* will denote the set of all worlds in which John left.

Interrogatives typically express a question. Although accounts of the meaning of questions differ in important ways, most of them agree that the meaning of a question is a set of alternatives: one alternative for each way that the question could be answered. Assuming that answers are propositions, if questions denote a set of answers, then questions denote a set of propositions, which for us will be a set of sets of worlds. For instance, *Did John leave?* will denote the set containing two elements: the set of worlds in which John left, and the set of worlds in which John didn't leave.

The third major clause type cross-linguistically is the imperative. Unlike declaratives and interrogatives, the denotation of an imperative is not settled. The most conservative hypothesis—conservative in the sense that it does not introduce machinery beyond what is already required for handling declaratives and questions—is that imperative denotations are propositions, plain and simple. This position has been defended in detail in M. Kaufmann 2011, and this approach has considerable initial plausibility:

(1) a. Sit down!
    b. You should sit down.

After all, both (1a) and (1b) can be used to command someone to sit down, so they presumably share at least some semantic content. Given that the deontic in (1b) can have a truth value, that is, can express a proposition, the simplest hypothesis is that (1a) can too. Since (1b) is true only if the addressee is under an obligation to sit down, a use of (1a) likewise guarantees an obligation to sit. The worry, of course, is that this approach attributes considerably more semantic structure to (1a) than is visible.

The next most conservative approach, in Portner (2005, 2007, 2009), is one step away from the imperatives-as-propositions view. He argues that imperatives denote properties, that is, functions from individuals to propositions. The meaning of *Sit down!* on this account is not a complete proposition, but a function that maps each potential addressee $x$ into the proposition that $x$ sat down. Uttering (1a) typically causes the addressee to place the sitting-down property on his or her To-Do list (a term of art for Portner). The To-Do list then induces an order on the set of future worlds, technically in exactly the way that Kratzer's 1991 ordering source does, and the dutiful addressee will behave only in ways that are compatible with some world that is ideal with respect to that ordering.

I explore here the possibility that imperatives denote actions. Then (1a) will denote a relation between worlds that differ by the sitting down of the addressee. If (1a) is uttered in some world $w$, then the sit-down action will relate $w$ to the set of worlds that reflect all the ways that the address might sit down: he might sit down in this chair, or he might sit down in that chair, and so on.

I am currently aware of three places in the linguistics literature in which actions have been proposed as the denotations of a natural language expression. Szabolcsi 1982 suggests that performatives in general denote actions. As she puts it, "[A]n act is something that brings about some change. Speech acts in general bring about changes in the speech situation... [P]erformative sentences should denote changes in the models... More precisely... the denotation of the sentence 'I congratulate you' at an index [world] *a* is a transition from *a* to another index [world] *b*." If imperatives such as (1a) are a type of speech act, Szabolcsi suggests that it denotes a set of transitions, that is, a relation over worlds.

Though not the same relation over worlds I am proposing. On Szabolcsi's analysis, a performance of (1a) would denote a relation that maps each utterance world onto a world in which the speaker has placed a new obligation on the addressee to sit down, not to the set of worlds in which the address sits down. The issue is whether the presumed agent of the action in question is the speaker (as in a performative) or the addressee (as in an imperative). Thus Szabolcsi's view builds the net pragmatic effect of the utterance into its meaning. For performatives such as promises or firings, this intermingling of utterance situation and content seems appropriate. On the view here, as we'll see, turning the expression of the sit-down action into an obligation requires some pragmatic work. I'm suggesting that there is a difference between *I hereby order you to sit down*, which is a performative in Szabolci's sense, and a bare imperative, such as *Sit down!*. In any case, Szabolcsi's analysis of performatives compellingly argues for the relevance of actions to the semantics of natural language.

In addition to Szabolcsi 1982, Naumann 2001 advocates actions as meanings. Specifically, he suggests that events should be modeled as actions. He proposes Dynamic Event Semantics, which he says is "based on the intuition that non-stative verbs express changes." Dynamic Event Semantics is based on Dynamic Modal Logic, which in turn is based on Dynamic Logic, which also serves as the starting point for the treatment of imperatives here.

Naumann's key idea is that instead of a plain Davidsonian event semantics

$$[\![ jump ]\!] = \lambda x[\lambda e.\mathbf{jump}(e,x)],$$

where the bracketed part is a function from events *e* to a proposition, we have

$$[\![ jump ]\!] = \lambda x[\lambda w.\mathbf{jump}'(w,x)],$$

where the bracketed part is an action, i.e., a function from a world *w* to the proposition (set of worlds) containing all those worlds that can be created from *w* by *x*'s jumping. There will be a world in which *x* jumps to the left, a world in which *x* jumps to the right, and so on. Then in place of traditional existential closure over events—which says that the evaluation world contains a suitable jumping event—we say instead that the evaluation world must be related to some successor world by the action of jumping.

There are many details that would need to be worked out to integrate a Naumannian view on events as actions into the current proposal, and I will not attempt to work through them here. I will mention, however, two strands of related research. Fernando 2002 seeks to decompose temporal relations denoted by event predicates into a sequence of causal changes (i.e., actions); and Pustejovsky and Moszkowicz 2011 use Dynamic Logic to model verbs of motion as sequences of updates that manipulate objects and their locations. The important point here is that we may need actions, or something very much like them, in order to build up the properties and propositions we need to compute for ordinary declarative sentence denotations. If so, then actions are independently-motivated semantical objects, and so readily available for analyzing imperatives and deontics.

The third place in the linguistics literature that advocates actions is Lascarides and Asher 2003, discussed below in section 5.

# 3 Imperatives as actions

I propose that imperatives, radically, simply, denote actions. As for Naumann, the basic meaning of an untensed action verb like *jump*, *sit*, *eat an apple* is a function from individuals to actions, so that, for instance, *John jump* is the action of John changing the world by performing a jumping event. Then an imperative is an action performed by the addressee: *Jump!* is the set of world pairs in which the second world is a continuation of the first world in which the addressee jumps.

There is no direct update effect. If uttering an imperative causes the addressee to believe that the speaker desires for an action to be performed, and if the addressee is inclined to fulfill the desires of the speaker, the imperative may influence the behavior of the addressee, not through grammatical regulation, but through simple pragmatical reasoning, very much in the way that thrusting a broom into the hands of an idle person and pointing at some dirt can cause the thrustee to behave as if they were newly under an obligation to begin sweeping.

## 3.1 Lack of truth conditions

Perhaps the simplest argument that imperatives do not denote propositions (at least, not directly) is that imperatives do not have truth conditions. Is *Sit down!* true or false? This is not a reasonable question.

Actions are relations over worlds, i.e., not propositions, so if imperatives denote actions, it is not surprising that imperatives do not have truth values. (Portner's imperative properties likewise fail to have truth values until they are applied to some argument, so this consideration may not distinguish between an action view and a property view.)

There is a tradition (Jørgensen's Dilema, see Hanson et al. 2007) under which deontics (*You may eat an apple*), being normative, can also be said to lack truth conditions. Van Rooij 2008 and Portner 2010 argue that the basic use of a deontic statement is to change what is permitted, i.e., that they are essentially performatives (in which case they denote actions by Szabolcsi's lights), and any truth-conditional descriptive use is derived from the performative meaning. I agree that descriptive meanings are built up from a non-propositional meaning (namely, from an action), though on the view here that basic meaning component is not performative. (See Barker 2010, section 7.2, for relevant discussion.)

At the very least, then, the meanings of imperatives and perhaps of some deontic expressions depend on elements that do not have truth conditions—so, perhaps on actions.

## 3.2 Only some predicates give rise to imperatives

All actions correspond to properties, but not all properties correspond to actions:

(2) a. Sit down! b. Jump!
(3) a. Don't sit down! b. Don't jump!

Sitting down and jumping are proper actions. Interestingly, so are their negations: refraining from sitting down, and refraining from jumping.

In contrast, stative properties are not suitable actions:

(4) a. *Be tall! b. *Be noticed! c. *Be lucky!
d. Get tall! e. Get noticed! f. Get lucky!

An essential component of an action is that the desired end situation must be result of a conscious, volitional choice on the part of the agent. To the extent that *get* differs from *be* in entailing volition, it can turn a stative property into an action. Getting tall, then, is what a teenager does by eating healthy food, getting noticed is what a person can do by dressing in an unusual manner, and getting lucky is what you can at least try to do by buying a lottery ticket.

Interestingly, sometimes the negation of a property can constitute an action if obeying the imperative requires willpower:

(5) a. *Be sad!
b. Don't be sad!

You can actively let yourself be sad, and you can remain sad, but the natural denotation of *be sad* is a non-action property. Adding negation will rescue a stative only in situations in which having the (positive) property is the default future, and volitional action can change that future (compare to **Don't exist*). Someone can tell you *Just be yourself* only if there is a reasonable expectation that you might choose to behave in a different way.

Passives also show that imperatives must have a volitional component:

(6) a. [to Mary] Kiss John!
b. [to John] *Be kissed by Mary!

(Though I note that Portner 2005 marks (6b) as grammatical.) Arguably, the two imperatives in (6) correspond to the same proposition, but only one expresses a proper (non-negated) action.

The distinction between actions and non-action properties can be easily managed if some predicates (e.g., *jump*) denote actions, which can be converted to properties or propositions as needed, and others (*tall*) denote (only) properties directly. Coercion principles would convert properties into actions when appropriate, as in the presence of *get* and, sometimes, negation.

## 3.3 Some predicates select for actions

One of the stronger arguments in favor of recognizing actions as a legitimate type of semantic object is that there are verbal and adjectival predicates that relate specifically to actions:

(7) a. Do something! b. Do nothing! c. Do everything Mary does!

Among the things you can do when you do something are jumping, sitting down, and laughing. It is difficult to understand not-jumping or not-sitting-down as doing something, so it appears that this use of *something* quantifies only over actions that are the denotation of some non-negated action description.

Likewise, the imperative *do nothing* will be fulfilled only if you choose the negation of a positive action. Refraining from not-jumping (i.e., choosing to jump) is not a legitimate way to do nothing. One way to understand this is to assume that the negative quantifier has scope over the verbal predicate, i.e., $\neg\exists a.\mathbf{do}(a)$, where $a$ ranges over positive actions.

If you do everything that Mary does, you must sit down if she sits down, and you must jump if she jumps. But you need not refrain from jumping if she refrains: doing everything Mary does does not entail not doing everything Mary doesn't do. Nor must you be tall if she is tall, nor must you seek to have the property of being identical to Mary. In other words, you need only mimic those situations involving Mary that are proper actions.

It is often said that quantifiers quantify only over atomic objects, and that this justifies distinguishing atoms from non-atomic sums in the ontology. Just so, if there are expressions that quantify only over actions, this justifies distinguishing actions from other properties in the ontology.

Finally, there are predicates such as *illegal* that apply only to actions, both positive (*It's illegal to smoke here*) and negative (*It's illegal not to report income*).

Just as the existence of predicates that apply only to kinds (e.g., *extinct*) or to non-atomic sums (*gather*) argue for recognizing kinds and non-atomic sums as distinct types of semantic object in their own right, so too with predicates that are specific to actions.

# 4 Ross' Paradox, and free choice permission

Let the preceding considerations justify considering a semantics in which imperatives at least, and perhaps some deontic expressions, have as an essential component of their meaning an action (a relation over worlds). The main empirical payoff will be a natural account of what is called Ross' Paradox for imperatives, and the problem of free choice permission for deontics.

Ross 1941 observed that adding alternatives to an imperative expands the range of behavior that an agent may safely perform.

(8) a. Slip the letter into the letter box.
    b. Slip the letter into the letter box or burn it.

Even though any action of slipping the letter into the letter box necessarily also counts as an action of either slipping the letter into the letter box or burning it, burning the letter is compatible only with the imperative in (8b) that explicitly mentions burning. We shall see shortly why the standard approach to modality makes the wrong prediction here.

Kamp 1973, 1978 recognized the importance of this problem for theories of the semantics/pragmatics interface in linguistic semantics. He concentrated on deontic declarative sentences, where the equivalent problem is known as the problem of free choice permission:

(9) a. You ate an apple or a pear.
    b. You ate an apple.
    c. You ate a pear.

If I see small black seeds on the kitchen table, I can assert (9a). Clearly, (9a) does not entail either (9b) or (9c). The inference patterns change radically in the presence of deontic modality:

(10) a. You may eat an apple or a pear.
     b. You may eat an apple.
     c. You may eat a pear.

Now there is an inference from (10a) to (10b), and also an inference from (10a) to (10c).

Zimmermann 2000 identifies disjunction as the critical element in the free choice implication here, and gives a radical proposal for the semantics of disjunction, suggesting that disjunction contributes a set of (for Zimmermann, exclusive, epistemic) alternatives.

## 4.1 How the standard modal semantics goes wrong

It is easiest to understand the nature of the problem by considering its deontic (free-choice) version.

On the proof-theoretic side, the standard modal theory extends classical logic with two modal operators, $\Box$ and $\Diamond$. In a deontic context, $\Box p$ means that $p$ is required, and $\Diamond p$ means that $p$ is permitted. It is assumed (though not in all of the systems described further below) that $\Diamond p$ if and only if $\neg\Box\neg p$ (something is permitted just in case it is not forbidden).

In addition to all of the classical axioms, all normal modal logics assume the principle of necessitation, which says that if $p$ is a theorem, $\Box p$ is a theorem. They all also adopt an axiom called K ('distribution'): $(\Box(p \to q)) \to ((\Box p) \to (\Box q))$. This minimal core common to all normal modal logics is called system **K** (after Saul Kripke).

In system **K**, the following is a theorem:

$$\text{Disaster:} \qquad (\Diamond p) \to \Diamond(p \vee q)$$

Technically, this theorem follows from the fact that $\Diamond p \to (\Diamond p \vee \Diamond q)$ (after all, if $p$ is possible, then certainly it's true that either $p$ is possible or that $q$ is possible) along with the fact that $(\Diamond p \vee \Diamond q) \equiv \Diamond(p \vee q)$. Hughes and Cresswell 1996:36 call this latter equivalence K6, and provide a detailed proof.

It's easy to see why this theorem is a disaster for modeling naturalistic permission talk:

(11) a. You may eat an apple. $\Diamond p$
 b. You may eat an apple or a pear. $\Diamond(p \vee q)$

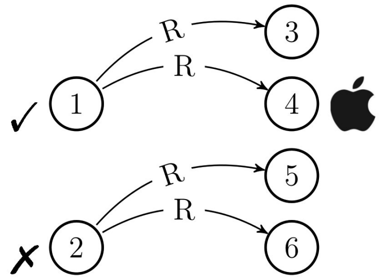
The disastrous theorem predicts that (11a) entails (11b). This runs violently counter to intuition: if you have permission to eat an apple, it does not follow at all that you therefore have permission to eat an apple or a pear.

Now, there may very well be some interpretation of (11b) on which it means 'You either have permission to eat an apple, or else you have permission to eat a pear (I either don't remember which, or am refusing for some reason to tell you)'. We can characterize this less-than-perfectly-informative interpretation as one on which disjunction takes scope over permission. The literature is in agreement that it is one possible reading for (11a). The problem is that standard modal logic predicts that this interpretation must be the only possible interpretation. But clearly (11b) also has an interpretation on which it gives strictly more permission than (11a), i.e., on which it is not entailed by (11a).

The standard denotational semantics for modal logic provides insight into the nature of the disastrous inference. On the standard modal logic account, the truth of a modal statement depends not only on what is happening in the world under consideration, but on what might be happening at that world. This is modeled by an accessibility relation $R$. For each world $w$, $R(w)$ returns the set of possible worlds that are in compliance with the laws of $w$.

(12) You may eat an apple.

Then (12) will be true at a world $w$ just in case at least one of the worlds accessible from $w$ is a world in which you eat an apple.




For instance, *You may eat an apple* is true at world $w_1$, because one of worlds that is deontically accessible from $w_1$ is a world in which you eat an apple (namely, world $w_4$). Apparently, then, eating an apple is consistent with obeying the rules as they are in $w_1$. The rules are different in world $w_2$, however, and since none of the worlds accessible from $w_2$ are apple-eating worlds, (12) is false at world $w_2$.

The disastrous inference arises from the fact that any world in which you eat an apple is also a world in which you eat an apple or a pear. This incorrectly predicts that (12) entails (13):

(13) You may eat an apple or a pear.

According to the semantic rule, (13) is predicted to be true if any of the accessible worlds is one in which you eat an apple or a pear. But any world in which you eat an apple is certainly a world in which you eat an apple or a pear; so once again, it is $w_4$ that guarantees the truth of (13). But this is wrong result: granting someone the permission expressed by (12) is not enough to permit pear eating.

So this is the problem of free choice permission.

In linguistics, the standard modal analysis has been superceded by Kratzer's (e.g., 1991) refinement, on which the traditional accessibility relation is decomposed into a more general accessibility relation (the modal base) along with a preference relation over worlds (the ordering source). Under Kratzer's theory, a world $w_4$ will only verify (12) if it is one of the maximally ideal worlds with respect to the ordering source. But this refinement will not help with the problem of free choice permission. The reason is quite simple: if (12) is true at $w_1$, then $w_4$ is ideal in the relevant sense. Since $w_4$ is still a world in which you eat an apple or a pear, $w_4$ still verifies (13) as well, and the undesired implication continues to go through.

As Zimmermann suggests, the lesson of these problems is that we need a non-classical understanding of disjunction.

# 5 Dynamic Logic

In the computer science literature and in the logical philosophical literature, there are a number of proposals to use actions to model imperatives, mostly stemming from Pratt's Dynamic Logic (see Pratt 1976, Harel 1984, Harel et al. 2000). Dynamic Logic was invented to reason about computer programs. Since the programming languages in question were highly imperative ("`print (x + 3)`"), it is no wonder that Dynamic Logic can be applied to reasoning about natural language imperatives. Segerberg 1990 is one of the better-known proposals to use Dynamic Logic for imperatives; see also Segerberg et al. 2009.

Within the linguistics literature, although Dynamic Logic is mentioned as one of the inspirations for Groenendijk and Stokhof's 1991 Dynamic Predicate Logic, as far as I know, only

Lascarides and Asher 2003 and Asher and Lascarides 2003 use actions in their formal semantics. (Starr 2010 also uses Dynamic Logic to give a semantics for imperatives, but on his approach, imperatives denote programs for updating preference relations, rather than for updating worlds, as we are exploring here.)

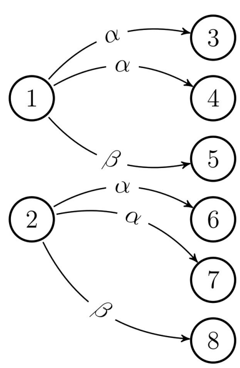
Lascarides and Asher's emphasis is on the need to update the common ground to reflect the content of imperatives: in the discourse *Go to the end of the road. There will be a large sign immediately in front of you*, the declarative will only be true in a context in which the imperative has been performed. As they put it 2003:6: "Semantically, the defining characteristic of a discourse which includes a commanded imperative is that its CCP changes the input world into an output one where the action has been performed." This may be putting the case too strongly, but it is certainly necessary to be in a position to update a context with the content of an imperative. If imperatives denote actions, gaining access to the updated worlds is a simple matter of taking the image of the context set under the relation denoted by the action, i.e., replacing each world in the common ground with the set of worlds resulting from the various ways in which the action in question could be performed.

DYNAMIC LOGIC (FRAGMENT)

| Actions: | States: | Inferences: |
|---|---|---|
| Atomic: $\alpha, \beta, \gamma, \ldots$ | Atomic: $A, B, C, \ldots$ | |
| $\alpha + \beta$, $\alpha;\beta$ | $A \vee B$, $A \wedge B$, $A \rightarrow B$, $\neg A$ | *All classical inferences, plus:* |
| | $[\alpha]A$, $\langle\alpha\rangle A \equiv \neg[\alpha]\neg A$ | $\dfrac{[\alpha+\beta]A}{[\alpha]A \wedge [\beta]A}$, $\dfrac{[\alpha;\beta]A}{[\alpha]([\beta]A)}$ |

'$[\alpha]A$' means 'Performing action $\alpha$ always results in state $A$'
'$\langle\alpha\rangle A$' means 'At least one way of performing action $\alpha$ results in state $A$'
'$[\alpha+\beta]A$' means 'Performing either action $\alpha$ or action $\beta$ results in $A$'
'$[\alpha;\beta]A$' means 'Performing $\alpha$, then $\beta$, results in $A$'

Denotations:
State: set of words
Action: relation over worlds
$[\![\alpha]\!] = \{\langle w_1, w_3\rangle, \langle w_1, w_4\rangle$
$\quad \langle w_2, w_6\rangle, \langle w_2, w_7\rangle\}$
$[\![\beta]\!] = \{\langle w_1, w_5\rangle, \langle w_2, w_8\rangle\}$
$[\![\alpha+\beta]\!] = [\![\alpha]\!] \cup [\![\beta]\!]$
$[\![\alpha;\beta]\!] = [\![\beta]\!] \circ [\![\alpha]\!]$




In this diagram, arrows labeled $\alpha$ show how a world changes when the addressee eats an apple; $\beta$ transitions involve eating a pear. If we start in world 1, eating one apple will take us to world 3, eating a different apple will take us to world 4, and eating the only pear will take us to world 5. Then $[\![$*eat an apple or a pear*$]\!] = [\![$*eat an apple*$]\!] \cup [\![$*eat a pear*$]\!] = \{\langle w_1, w_3\rangle, \langle w_1, w_4\rangle, \langle w_1, w_5\rangle, \langle w_2, w_6\rangle, \langle w_2, w_7\rangle\langle w_2, w_8\rangle\}$. In this situation, the imperative *Eat an* *apple or a pear* gives strictly more options for action than *Eat an apple*, as desired. This gives a satisfying account of Ross' Paradox.

Likewise, on a deontic interpretation, *you may eat an apple* asserts that the set of worlds at the end of apple eating arcs are within the sphere of permissibility ($[\alpha]\text{OK}$). Since *you may eat an apple or a pear* guarantees that a larger set of worlds is permissible, we have a satisfying account of Free Choice Permission.

In a more realistic model of how actions change the world, the set of arcs will be constrained by world knowledge (eating an apple can only occur in a world in which there is an apple available), and by grammatical patterns (the action denotes by *eat an apple* will be a subset of the action denoted by *eat a piece of fruit*).

The notation $[\alpha]A$ is intended to look like a necessity box decorated with an action. Instead of trying to make do with a single comprehensive accessibility relation $R$, Dynamic Logic supplies a distinct flavor of accessibility relation corresponding to each distinct action.

I should mention that there is an epistemic variant of the problem of free choice: from *John might be in London or in Paris*, infer that John might be in London, and also that John might be in Paris. In order to extend the approach suggested here to epistemic cases, we would have to conceive of some actions as relating epistemic states rather than as relating worlds. The result might look very much like the treatment of epistemic free choice in Ciardelli, Groenendijk and Roelofsen 2011, or perhaps as in Starr 2011.

## 5.1 Resource-sensitivity: tracking the amount of permission

In Barker 2010, following Lokhorst 1997, 2006, I diagnose the problems that arise from using classical logic for deontics as a lack of resource sensitivity. Classically, if *You may eat an apple or a pear* entails both *You may eat an apple* and *You may eat a pear*, it also guarantees that you are allowed to eat both an apple and a pear. This gives too much permission. Fox 2007 has the opposite problem: on his proposal of recursive exhaustification, deriving the free choice implication also guarantees that the addressee is prohibited from eating both an apple and a pear; but that is too restrictive. As Simons 2005 notes, saying *You may eat an apple or a pear* must neither guarantee permission to eat both, nor prohibit it. (Interestingly, Franke's 2011 game-theoretic approach provides the appropriate degree of flexible in this respect.)

In any case, the Dynamic Logic approach explored here is resource-sensitive in the relevant sense. In particular, the action of eating an apple or a pear is not the same action as eating an apple and then eating a pear (i.e., $\alpha + \beta \neq \alpha;\beta$). Permission to perform the disjunctive action, then, definitely gives permission to eat an apple, and definitely gives permission to eat a pear (the free choice permission problem), but says nothing about whether it's ok to eat both.

## 5.2 The problem with negating actions

There are at least two ways to add negation to Dynamic Logic, and it is not obvious whether either is appropriate for natural-language imperatives or deontics. As indicated in the fragment above, there is negation of states: $\neg[\alpha]A$ says that it is not the case that performing action $\alpha$ guarantees state $A$. This is clear enough. But what about negating an action? What should $[\neg\alpha]A$ mean? The negation of an action can either be thought of in terms of the elements in the relation denoted by the action, in which case $\neg\alpha$ is the relation that holds between two worlds $\langle w_1, w_2\rangle$ just in case $\langle w_1, w_2\rangle \notin \alpha$; or it can be thought of in terms of results, in which case $\neg\alpha$ is the relation that holds between two worlds $\langle w_1, w_2 \rangle$ just in case $w_2 \notin A$. It's not clear that either approach is particularly appropriate when applied to natural language (though see remarks below concerning the embedding into Linear Logic).

## 5.3 Weak permission versus strong permission

Just as the resource-sensitivity of Dynamic Logic enables it to calibrate the precise amount of permission, it also enables it to distinguish between weak permission and strong permission (see, e.g., Hansen 2007 for the distinction between weak and strong permission). If something is merely not prohibited, that is weak permission. But if something is explicitly allowed, that is a permissive norm, i.e., strong permission. In classical modal logic, as mentioned above, $\Diamond p$ if and only if $\neg\Box\neg p$, so weak permission is the only kind of permission there is.

There is some reason to think that permissive sentences express strong permission (Asher and Bonevac 2005). After all, if I tell you *You may eat an apple*, then apple-eating is explicitly permitted. And if I tell you *You may not eat a pear*, then pear-eating is explicitly forbidden. But then what about eating bananas? It is not forbidden, but that does not automatically put it in the same category as apple-eating. In natural language, as in real life, it is not the case that whatever isn't forbidden is permitted.

Koji Mineshima (personal communication) suggests that Dynamic Logic can distinguish weak permission from strong permission by distinguishing between $[\alpha]A$ (every way of performing action $\alpha$ is ok) from $\langle\alpha\rangle A$ (at least one way of performing $\alpha$ is ok, i.e., it is not the case that $\alpha$ is forbidden). Then the free choice interpretation of *You may eat an apple* is strong permission ($[\alpha]\text{OK}$), and the normal interpretation of a negated permissive (*You may not eat an apple or a pear*) is the negation of weak permission: $\neg\langle\alpha\rangle\text{OK} \equiv [\alpha]\neg\text{OK}$.

## 5.4 The (dis)unity of disjunction

The fragment above gives two distinct interpretations for disjunction: $\vee$, disjunction of states (propositions), corresponding to union over sets of worlds; and $+$, disjunction of actions (relations over worlds), corresponding to union over sets of pairs of worlds. In both cases, disjunction corresponds to the union operation, which seems intuitive enough; yet it remains union over two different semantic domains. Is *or* polysemous in this way?

Typologically, languages often distinguish among two or more notions of disjunction. The difference between inclusive and exclusive uses is rarely (if ever) grammatically encoded, but many languages distinguish between what Mauri 2008 calls a 'standard' meaning for disjunction versus an 'interrogative' (irrealis) meaning. It is not clear how (or even whether) this distinction aligns with a semantic distinction between propositions versus actions (interrogative?), but it is important to carefully evaluate approaches according to whether they assume that disjunction is a unitary concept or polysemous.

## 5.5 Comparison with Barker's 2010 Linear Logic account

The Linear Logic account of permission advocated in Barker 2010 (following Lokhorst 1997, 2006) does not make an ontological distinction between propositions and actions, which both correspond to sets of evaluation points (which are better conceived of as information states that as worlds). Propositions have the special property that they are idempotent with respect to the tensor operation, i.e., stable under (multiplicative) conjunction with themselves. That is, the truth of the claim that *it is raining* is equivalent to the truth of *it is raining and it is raining*. In contrast, the action expressed by *eat an apple* is a different action from that expressed by *eat an apple and eat an apple*—only the second requires more than one piece of fruit.

In addition, since negation in Linear Logic is defined for any formula, negation extends smoothly from propositions to actions.

The unity of *or* and the generality of negation seem to give Linear Logic conceptual advantages over the Dynamic Logic approach.

On the other hand, standard Linear Logic is commutative. In the application to imperatives and deontics, this incorrectly predicts that *Cook dinner and do the dishes* necessarily expresses the same complex action as *Do the dishes and cook dinner*. In contrast, on the Dynamic Logic account, $\alpha;\beta \neq \beta;\alpha$, since sequencing of actions (i.e., composition) is not commutative: traversing an $\alpha$ arc and then traversing a $\beta$ arc will in general take you to a very different set of worlds than first traversing a $\beta$ arc and then traversing an $\alpha$ arc. As far as I know, there is no consensus about the most natural way to construct a non-commutative version of Linear Logic, let alone a consensus as to whether such a logic is well-suited to modeling natural language. However, Abrucia and Ruet 1999 propose a non-commutative refinement of Linear Logic along the lines of Lambek's original resource-sensitive logic for natural language, which can serve as a starting point.

Another striking difference between Dynamic Logic and Linear Logic is the simplicity and clarity of the semantics for Dynamic Logic, compared with the opacity of the semantics for Linear Logic. Some insight may be gained by extending standard methods for embedding classical logic in Linear Logic (e.g., Lafont 1999) along lines suggested by Koji Mineshima (personal communication) to provide an embedding of Dynamic Logic into Linear Logic on which a permissive statement such as *You may eat an apple* $[\alpha]A$ translates as $\alpha \multimap A$. Then disjunction of both propositions ($\vee$) and actions ($+$) translate uniformly as additive disjunction $\oplus$, and $\neg[\alpha]A$ translates as $(\alpha \multimap A)^{\perp}$.

# 6 Conclusions

Actions (relations over worlds) are indispensable elements in a complete semantic analysis of natural language: they are essential for understanding performatives (Szabolcsi), they may be a superior way to conceive of event semantics (Naumann); and they allow computing the update effect of a discourse containing imperatives (Lascarides and Asher). In addition, they provide a radically minimal account of imperatives on which imperatives denote bare actions, pure and simple. This captures the resources-sensitivity of imperatives and deontics, notably their non-commutativity, and gives a satisfying account of Ross' Paradox and free choice permission.

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