
Scope

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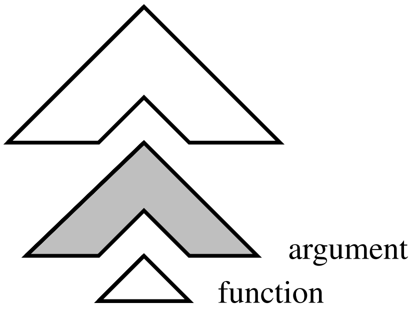
1 Scope basics

Scope-taking is one of the most fundamental, one of the most characteristic, and one of the most dramatic features of the syntax and semantics of natural languages.

A phrase **takes scope** over a larger expression that contains it when the larger expression serves as the smaller phrase's semantic argument.

(1) John said [**Mary called** [everyone] **yesterday**] with relief.

The following diagram schematizes the scope-taking illustrated in (1):



In this picture, the context *John said* [ ] *with relief* corresponds to the upper unshaded notched triangle, the embedded context *Mary called* [ ] *yesterday* corresponds to the middle grey notched triangle, and the scope-taker *everyone* corresponds to the lower unshaded triangle.

In (1), *everyone* takes scope over the rest of the embedded clause that surrounds it, namely, *Mary called* [ ] *yesterday*. What this means semantically is that *everyone* denotes a function that takes as its argument the property corresponding to the surrounding embedded clause with the position occupied by the scope-taker abstracted, namely, $\lambda x.\mathbf{yesterday}(\mathbf{called}\ x)\ \mathbf{m}$. I will call the expression over which the scope-taker takes scope (the grey region in the diagram) its **nuclear scope**.

1.1 The difference between scope and quantification

There is a close and non-accidental correspondence between scope-taking and quantification. Quantifiers construct a meaning by considering alternatives one by one. That is, *Mary called everyone yesterday* is true just in case for every choice of a person x, substituting x in place of *everyone* leads to a true proposition. When a quantifier appears in an embedded argument position (as *everyone* does in *Mary called everyone yesterday*), the only way for it to gain access to the predicate it needs is by taking scope. So some quantifiers are forced by the nature of their meaning and their syntactic position to take scope.

Revised draft. Thanks to Lucas Champollion, Simon Charlow, Jeremy Kuhn, Mike Solomon, Anna Szabolcsi, and the handbook editors. Some conventions: Semantic values associate to the left, so that $fab \equiv (fa)b$, and semantic types associate to the right, so that $\mathsf{a} \rightarrow \mathsf{b} \rightarrow r \equiv \mathsf{a} \rightarrow (\mathsf{b} \rightarrow r)$.

Some of the many quantificational expressions that arguably require (non-trivial) scope include quantificational DPs (e.g., *everyone*), quantificational determiners (*every*), quantificational adverbs (*mostly*), adjectives (*occasionally*, *same* and *different*), and comparatives and superlatives (*–er*, *–est*).

But in general, scope and quantification are logically independent. On the one hand, there are expression types that are quantificational but that occur in predicate position, and so do not need to take scope. These include tense, modal auxiliaries, dynamic negation, etc. On the other hand, there are expressions that arguably take displaced scope, but which are not necessarily quantificational, such as question particles, wh-words, disjunction, some analyses of proforms (both overt and silent), expressives such as *damn*, etc.

1.2 Some additional resources

There are many excellent discussions of scope. I will mention only four here. The article by Westerståhl in this volume ('Generalized Quantifiers') complements the current article closely, addressing a number of issues relating to scope not discussed here, notably an innovative treatment of the scope of possessives based on Peters & Westerståhl (2006). Ruys & Winter (2011) and Steedman (2012) discuss many of the phenomena and issues treated here in some depth. Finally, Szabolcsi (2010) is an indispensable resource on quantification and on scope in English and many other languages.

1.3 Scope ambiguity

If a scope-taking element can take scope in more than one way, a sentence that contains it may be ambiguous as a result.

(2) a. Ann intends to marry each man she meets.
b. *Each* takes wide scope over *intend*: For each man *x*, Ann intends to marry *x*.
c. *Intend* takes wide scope over *each*: Ann intends for her marriage partners to exhaust the set of men that she meets.

The modal verb *intends* does not take special scope, always taking just its syntactic complement as its argument. But the quantifier *each man* can take scope over the embedded infinitival, or over the entire sentence. This indeterminacy creates semantic ambiguity: (2a) either has the interpretation given in (2b), on which Ann forms attachments easily, though she may also have an intention of only ever marrying at most one person. The second interpretation describes a more ambitious person, one who sets out to marry a potentially large set of men.

If there is more than one scope-taking element in the sentence, it often happens that either one can take wide scope:

(3) a. A man ate every cookie.
b. **Linear scope**: *a* outscopes *every*: There is a man who ate every cookie.
c. **Inverse scope**: *every* outscopes *a*:
For every cookie *x*, there is some potentially different man who ate *x*.

The standard assumption is that this ambiguity is purely semantic in nature, and should be explained by the same mechanism that gives rise to scope-taking.

Note that the reading in (3b) entails the reading in (3c). Entailment relations among different scopings are common.

(4) Every woman saw every man.

In fact, when both scope-taking elements are universal quantifiers (likewise, when both are indefinite determiners), there is an entailment relation in both directions, so that the readings are indistinguishable from the point of view of truth conditions: whether we check for every woman whether every man saw her, or check for every man whether he was seen by every woman, we arrive at the same set of seeing events. The two readings still correspond to clearly distinct meanings, although the sentences are true in the same class of situations.

1.4 Linear scope bias

The more prominent reading of the sentences in (3) and (4) correspond to the linear order of the quantifiers in the sentence. The preference for linear scope is robust across construction types and across languages. In addition, if any scoping is available, at least the linear scoping will certainly be available.

1.5 Inverse scope versus inverse linking

Sometimes a DP embedded inside of another DP can take wide scope with respect to the host DP.

(5) a. [Some person from [every city]] loves it.
  b. There is a person who is from every city and who loves some salient thing.
  c. For every city *x*, there is some person *y* who is from *x*, and *y* loves *x*.

In (5), there are two scope interpretations. On the first interpretation, there is some person who has lived in each of some salient set of cities. On the second interpretation, for each choice of a city, there must be some (potentially different) person who is from that city.

This second reading is similar to inverse scope, but distinct from it. It is known as the **inverse linking** reading (May (1977, 1985); May & Bale (2005)), and it is often more prominent than the non-inversely linked reading (when the latter is available at all). Although the inverse linking reading gives wide scope to the quantifier whose determiner (here, *every*) linearly follows the determiner that heads the other quantifier (*some*), this is not a counterexample to the linear scope bias, since the linear scope bias concerns quantifiers that follow one another, and in (5), one quantifier is contained within the other, as shown by the brackets in (5a). Inverse linking is sporadic; for instance, there is no inverse linking reading of *no one from no city*, which would otherwise have a reading equivalent to (5c). Note that in (5), the universal quantifier is able to bind the pronoun in the verb phrase only under the inverse linking reading.

1.6 Scope islands

Not all logically possible scope relations are grammatical.

(6) a. Someone thought [everyone left].
  b. There is a person who thought that everyone left.
  c. For each person x, there is some person y such that y thought x left.

Native speakers report that only (6b) is a possible paraphrase of (6a). In other words, the universal quantifier embedded inside the bracketed clause cannot take scope over the quantifier in matrix subject position. In English, tensed clauses are generally thought to be **scope islands** for universal quantifiers. For at least some speakers, infinitival clauses are not scope islands, so that *Someone asked everyone to leave* can be ambiguous. Some speakers allow the universal quantifier *each* to scope out of some tensed clauses (Szabolcsi (2010):107).

Relative clauses are particularly strong scope islands.

(7) a. A woman from every borough spoke.
  b. A woman [who is from every borough] spoke.

There is an inverse-linking reading for (7a) on which the universal takes wide scope relative to the indefinite, so that there are potentially as many women who spoke as there are boroughs. But the bracketed relative clause in (7b) is a scope island for *everyone*, and therefore is unambiguous: there must be a single woman such that for every borough, the woman is from the borough. This property makes relative clauses useful for constructing unambiguous paraphrases of scopally ambiguous sentences.

Scope islands are sensitive to the identity of the scope-taking element in question. In particular, indefinites are able to escape from any scope island, as discussed in section 5.

1.7 Scope and ellipsis

Quantifier scope interacts with ellipsis in ways that have been argued to constrain both the theory of scope-taking and the theory of ellipsis.

(8) a. A woman watched every movie, and a man did too.
  b. A woman watched every movie, and Mary did too.

In the verb phrase ellipsis example in (8a), the left conjunct is interpreted as if the missing verb phrase were *watched every movie*. But of course, the unelided sentence *a man watched every movie* is ambiguous with respect to linear scope versus inverse scope. Either scoping interpretation is possible, as long as the interpretation of the first conjunct is parallel. That is, (8a) can be interpreted with linear scope for both conjuncts, or with inverse scope for both conjuncts, but mismatched scope relations across the conjuncts are not allowed. One way to think of this informally is that the antecedent clause decides what scoping it prefers, and then the ellipsis process copies that preference to the elided clause.

However, when the indefinite subject of the elided VP is replaced with a proper name, as in (8b), the ambiguity disappears. According to Fox (2000), this is due to general considerations of **derivational economy**, which allow a quantifier to take inverse scope only if doing so has a detectable effect on truth conditions. Taking inverse scope over a proper name like Mary has no effect on truth conditions, so Economy limits the interpretation of the elided VP to linear scope; and the fact that the scope of the ellipsis clause must match the scope of its antecedent limits the interpretation of the left conjunct to the only scoping that is consistent with Economy in the second clause. See Johnson & Lappin (1997, 1999) for a critique of Economy, including a discussion of scope.

The sluicing example in (9) is also unambiguous, though for a different reason.

(9) A woman watched every movie, but I don't know who.

As discussed in Barker (2013), the indefinite *a woman* in the antecedent clause can only serve as the wh-correlate if it takes scope over the rest of the antecedent clause.

2 Theories of scope

The basic challenge for any theory of scope-taking is to explain how it is possible for a scope-taker to reverse the normal direction of function/argument composition, in order to provide the scope-taking element with access to material that properly surrounds it.

The theories discussed here are Quantifying In, Quantifier Raising, Cooper Storage, Flexible Montague Grammar, Scope as surface constituency (Steedman's combinatory categorial grammar), type-logical grammar, the Lambek-Grishin calculus, and Discontinuous Lambek Grammar. A discussion of the continuation-based system of Shan & Barker (2006) and Barker & Shan (2008) is postponed until section 3.

2.1 Quantifying In

The historically important Montague (1974) proposes a generative grammar in which scope-taking is managed by two more or less independent systems. The first system is an in-situ strategy on which verbs and other predicates denote relations over generalized quantifiers (where extensional quantifiers have type $(\mathtt{e} \to \mathtt{t}) \to \mathtt{t}$), rather than over individuals (type $\mathtt{e}$). As a result, unlike systems such as Quantifier Raising (see next subsection), there is no type clash when a quantificational DP occurs in argument position. However, given only the in-situ strategy, the scope domain of a quantifier is limited to the functional domain of the predicate that takes it as an argument. Furthermore, the account of scope ambiguity is insufficiently general, since scope relations are fully determined by the lexical meaning of the predicates involved.

These deficiencies in the in-situ scope mechanism are addressed by the other scope-taking system, which involves an operation called Quantifying In (QI). Quantifying In provides for scope domains of unbounded size, and also accounts for scope ambiguity independently of lexical meaning. Syntactically, QI replaces the leftmost occurrence of a pronoun with the quantifier phrase. The corresponding semantic operation abstracts over the variable denoted by the pronoun, and delivers the resulting property to the quantifier to serve as the quantifier's semantic argument.

Syntax: QI$_{\text{SYN}}$(everyone, [John [called he]]) = [John [called everyone]].
Semantics: QI$_{\text{SEM}}$(**everyone**, **called** x **john**) = **everyone**($\lambda x.$(**called** x **john**))

The quantifier does not enter the derivation until its entire scope domain has been constructed. This allows the quantifier to take its scope domain as a semantic argument in the normal way, at the same time that the quantifier appears syntactically in a deeply embedded position within its nuclear scope.

Quantifier scope ambiguity is explained by quantifying into the same phrase structure in different orders: quantifiers that undergo quantifying-in later take wider scope than those that undergo QI earlier.

2.2 Quantifier Raising

By far the dominant way to think about scope-taking is Quantifier Raising (QR), as discussed in detail in May (1977), Heim & Kratzer (1998), and many other places. QR is in some sense the inverse of the quantifying-in operation just described.

In Quantifier Raising, the quantifier combines (merges) syntactically in the embedded position in which it appears on the surface. The operation of Quantifier Raising moves the quantifier to adjoin to its scope domain, placing a variable in the original position of the quantifier, and abstracting over the variable at the level of the scope domain.

$$[\text{John [called everyone]}] \overset{\text{QR}}{\Rightarrow} [\text{everyone}(\lambda x[\text{John [called } x]])]$$

Here, the scope domain of *everyone* is the entire clause. The structure created by QR is known as a Logical Form.

Because the sentence is pronounced using the word order before QR has occurred, QR is thought of as 'covert' (invisible) movement (though see Kayne (1998) for an analysis on which scope-taking is *overt* movement). For comparison with a standard example of overt movement, consider the wh-fronting that occurs in some embedded questions, such as the bracketed phrase in *I know* [*who (λx. John called x)*]. In this case, the pronounced word order (in English) reflects the position of the scope-taking element (here, the wh-phrase *who*) after it has been displaced by movement.

One standard presentation of Quantifier Raising is Heim & Kratzer (1998). They point out that when a quantifier appears in, say, direct object position, as in the example above, there is no mode of semantic combination (certainly not function application) that allows the meaning of the verb to combine directly with the meaning of the quantificational direct object. Then Quantifier Raising is motivated as one way to rescue this kind of type clash.

Precisely because there is an otherwise unresolvable type clash before QR, in the terminology of, e.g., Jacobson (2002), the QR strategy fails to be 'directly compositional'. The reason is that there is a level of analysis at which a well-formed syntactic constituent fails to have a correspondingly well-formed semantic analysis, e.g., in the verb phrase *called everyone* in the pre-QR structure given above.

QR easily accounts for inverse scope by allowing QR to target quantifiers in any order.

$$\begin{array}{rl}
\text{Linear scoping :} & \text{[someone [called everyone]]} \\
& \overset{\text{QR}}{\Rightarrow} \text{[everyone(}\lambda x\text{[someone [called } x\text{]])]} \\
& \overset{\text{QR}}{\Rightarrow} \text{[someone(}\lambda y\text{[everyone(}\lambda x\text{[}y\text{ [called } x\text{]])])]} \\
\text{Inverse scoping :} & \text{[someone [called everyone]]} \\
& \overset{\text{QR}}{\Rightarrow} \text{[someone(}\lambda y\text{[}y\text{ [called everyone]])]} \\
& \overset{\text{QR}}{\Rightarrow} \text{[everyone(}\lambda x\text{[someone(}\lambda y\text{[}y\text{ [called } x\text{]])])]}
\end{array}$$

Raising the direct object first and then the subject gives linear scope, and raising the subject first and then the direct object gives inverse scope.

QR also easily accounts for inverse linking, in which a quantifier embedded inside of a quantificational DP takes scope over the enclosing DP:

$$\begin{array}{rl}
\text{Inverse linking:} & \text{[[some [friend [of everyone]]][called]]} \\
& \overset{\text{QR}}{\Rightarrow} \text{[[some [friend [of everyone]]](}\lambda x\text{[}x\text{ called])]} \\
& \overset{\text{QR}}{\Rightarrow} \text{[everyone(}\lambda y\text{[[some [friend [of } y\text{]]](}\lambda x\text{[}x\text{ called])])]}
\end{array}$$

In some accounts (May (1985); Barker (1995); Büring (2004)) DP is a scope island, and the embedded quantifier cannot take scope outside of its host DP. See Sauerland (2005) for an opposing view, and Charlow (2010) for discussion.

Care is needed, however, to prevent a sequence of QR operations from leaving an unbound trace:

$$\begin{array}{rl}
\text{Unbound trace:} & \text{[[some [friend [of everyone]]][called]]} \\
& \overset{\text{QR}}{\Rightarrow} \text{[everyone(}\lambda y\text{[[some [friend [of } y\text{]]][called]])]} \\
& \overset{\text{QR}}{\Rightarrow} \text{[[some [friend [of } y\text{]]](}\lambda x\text{[everyone(}\lambda y.x\text{)][called])]}
\end{array}$$

If QR targets the embedded quantifier *everyone* first, and then targets the originally enclosing quantifier *some friend of _*, the variable introduced by the QR of *everyone* (in this case, y) will end up unbound (free) in the final Logical Form structure. Such derivations must be stipulated to be ill-formed.

2.3 Cooper Storage

For both Quantifying In and Quantifier Raising, it is necessary to construct (parse) the entire nuclear scope before the quantifier can take scope. Cooper (1983) proposes building structures from the bottom up in a way that does not require waiting.

Here is how it works: when a quantifier is first encountered, a pronoun is placed in the position of the quantifier, and the quantifier (along with the index of the pronoun) is placed in a multiset (i.e., an unordered list) that is kept separate from the syntactic structure. The list of quantifiers is called the **store**.

Syntactic parsing and semantic composition proceeds upwards, building two separate structures in parallel: a tree structure (along with its semantic interpretation) consisting of the non-quantificational elements of the sentence, and a list of quantifiers that have been encountered so far. At the point at which a quantifier can take scope (typically, a clause node), the quantifier is removed from the store, the associated index is used to abstract over the placeholder pronoun, and the quantifier takes the resulting property as its semantic argument. A derivation is complete only when the store is empty, i.e., only when all of the quantifiers have been scoped out.

SYNTAX	SEMANTICS	STORE
1. called everyone	$\mathbf{call}\ x$	$[\langle \mathbf{e'one}, x\rangle]$
2. someone [called everyone]	$\mathbf{call}\ x\ y$	$[\langle \mathbf{e'one}, x\rangle, \langle \mathbf{s'one}, y\rangle]$
3. someone [called everyone]	$\mathbf{s'one}(\lambda y.\mathbf{call}\ x\ y)$	$[\langle \mathbf{e'one}, x\rangle]\rangle$
4. someone [called everyone]	$\mathbf{e'one}(\lambda x.\mathbf{s'one}(\lambda y.\mathbf{call}\ x\ y))$	$[]$

The syntactic structure is built up in steps 1 and 2. The subject quantifier is removed from the store in step 3, and the object quantifier is removed in step 4, at which point the store is empty and the derivation is complete. Since the store is unordered, quantifiers can be removed in any order, accounting for scope ambiguity.

Cooper storage is mentioned below in the discussion of semantic underepresentation in section 8.

2.4 Flexible Montague Grammar

Hendriks's (1993) Flexible Montague Grammar accounts for a wide variety of scope-taking configurations using two main semantic type-shifting rules, Argument Raising and Value Raising. (Hendriks discusses two other type-shifting rules that I ignore here.)

Argument Raising gives the *i*th argument of a predicate wide scope over the predicate and the rest of its arguments.

Argument Raising (AR): if an expression ϕ has a denotation

$$\lambda x_1 \lambda x_2 ... \lambda x_i ... \lambda x_n [f(x_1, x_2, ..., x_i, ..., x_n)]$$

with type

$$a_1 \to a_2 \to ... \to a_i \to ... \to a_n \to r,$$

then ϕ also has the denotation

$$\lambda x_1 \lambda x_2 ... \lambda x_i ... \lambda x_n [x_i(\lambda x.f(x_1, x_2, ..., x, ..., x_n))]$$

with type

$$a_1 \to a_2 \to ... \to ((a_i \to r) \to r) \to ... \to a_n \to r.$$

In order to model the two scopings of *Someone saw everyone*, we need to apply Argument Raising to the verb *saw* twice. Let $\mathtt{G}$ be the type of an extensional generalized quantifier, i.e., $\mathtt{G} \equiv (\mathtt{e} \to \mathtt{t}) \to \mathtt{t}$:

$$\begin{array}{ccccc} \mathtt{e} \to \mathtt{e} \to \mathtt{t} & \text{AR} & \mathtt{G} \to \mathtt{e} \to \mathtt{t} & \text{AR} & \mathtt{G} \to \mathtt{G} \to \mathtt{t} \\ \text{saw} & \Rightarrow & \text{saw} & \Rightarrow & \text{saw} \\ \lambda xy.\mathbf{saw}\, x\, y & & \lambda \mathcal{X} y.\mathcal{X}(\lambda x.\mathbf{saw}\, x\, y) & & \lambda \mathcal{X}\mathcal{Y}.\mathcal{Y}(\lambda y.\mathcal{X}(\lambda x.\mathbf{saw}\, x\, y)) \end{array}$$

When the doubly-type-shifted denotation for *saw* combines first with *everyone* and then with *someone*, the second argument (syntactically, the subject) takes scope over the first argument (the direct object), giving linear scope. If we had applied Argument Raising in the opposite order (i.e., raising the type of the second argument before raising the type of the first), we would have the same final type, but the new denotation would exhibit the other scoping, namely $\lambda \mathcal{X}\mathcal{Y}.\mathcal{X}(\lambda x.\mathcal{Y}(\lambda y.\mathbf{saw}\, x\, y))$, corresponding to inverse scope. Despite the reversal of the scope relations, both shifted versions of the verb combine with their two arguments in the same order: first with the direct object, and then with the subject. The difference in interpretation arises from the order in which the type $\mathtt{e}$ argument positions of the underlying relation (represented by the variables x and y) are abstracted over in order to compose with the generalized quantifiers.

The second main type-shifting rule, Value Raising, allows expressions to take scope wider than their local functor.

Value Raising (VR): if an expression ϕ has a denotation

$$\lambda x_1 ... \lambda x_n [f(x_1, ..., x_n)] \text{ with type } a_1 \to ... \to a_n \to r,$$

then for all types r', ϕ also has the denotation

$$\lambda x_1 ... \lambda x_n \lambda \kappa [\kappa(f(x_1, ..., x_n))] \text{ with type } a_1 \to ... \to a_n \to (r \to r') \to r'.$$

For instance, Value Raising allows a quantifier such as *everyone* to scope out of possessor position, as in *Everyone's mother left.* Assume that the basic type of the relational noun *mother* is a function of type $\mathtt{e} \to \mathtt{e}$ mapping people to their mothers. Then in addition to its basic type, *mother* will have the following shifted types:

$$\begin{array}{ccccc} \mathtt{e} \to \mathtt{e} & \text{VR} & \mathtt{e} \to \mathtt{G} & \text{AR} & \mathtt{G} \to \mathtt{G} \\ \text{mother} & \Rightarrow & \text{mother} & \Rightarrow & \text{mother} \\ \lambda x.\mathbf{mom}\, x & & \lambda x \kappa.\kappa(\mathbf{mom}\, x) & & \lambda \mathcal{P} \kappa.\mathcal{P}(\lambda x.\kappa(\mathbf{mom}\, x)) \end{array}$$

The doubly-shifted *mother* can serve as a modifier of the generalized quantifier *everyone*, allowing it to combine with and take scope over an Argument-Raised version of *left*:

$$\begin{aligned}[\![\mathit{left}]\!]([\![\mathit{mother}]\!][\![\mathit{everyone}]\!]) &= (\lambda\mathcal{P}.\mathcal{P}\mathbf{left})((\lambda\mathcal{P}\kappa.\mathcal{P}(\lambda x.\kappa(\mathbf{mom}\ x)))\ \mathbf{everyone})\\ &= \mathbf{everyone}(\lambda x.\mathbf{left}(\mathbf{mom}\ x))\end{aligned}$$

In combination with Argument Raising, Value Raising allows scope-takers to take scope over an arbitrarily large amount of surrounding context.

Unlike Quantifier Raising, these type-shifting rules do not disturb syntactic categories or syntactic constituency in the slightest. In this sense, then, Flexible Montague Grammar captures the intuition that scope-taking amounts to covert movement.

However, a Flexible Montague Grammar semantic translation is only well-defined if the semantic type of each argument matches the semantic type expected by its functor. Thus the grammar must have two levels of well-formedness checking: a syntactic level of function/argument composition, and a semantic level making sure that the type of each (possibly shifted) argument matches that of its (possibly shifted) functor.

One peculiar feature of Flexible Montague Grammar is that since the type-shifters operate only on predicates, the system locates scope taking and scope ambiguity entirely in the verbal predicates, rather than in the quantifiers themselves, or in some more general aspect of the formal system.

Although conceptually elegant, in practice Flexible Montague Grammar is somewhat cumbersome, and full derivations are rarely seen.

2.5 Function composition: scope as surface constituency

Steedman (2012):110 offers a combinator-based grammar that addresses quantifier scope. Among the lexical entries generated by his system for *everyone* and for *no one* are the following:

(10) a. everyone$_a$ $\mathrm{s}/(\textsc{dp}\backslash \mathrm{s})$ $\lambda\kappa\forall x.\kappa x$
b. everyone$_b$ $((\textsc{dp}\backslash \mathrm{s})/\textsc{dp})\backslash(\textsc{dp}\backslash \mathrm{s})$ $\lambda\kappa y\forall x.\kappa x y$
c. no one$_c$ $(\mathrm{s}/\textsc{dp})\backslash \mathrm{s}$ $\lambda\kappa\neg\exists x.\kappa x$
d. no one$_d$ $((\textsc{dp}\backslash \mathrm{s})/\textsc{dp})\backslash(\textsc{dp}\backslash \mathrm{s})$ $\lambda\kappa y\neg\exists x.\kappa x y$

I have recast Steedman's notation to conform to the Lambek/type-logical tradition, in order to match the convention used throughout the rest of this article. In the Lambek style, the argument category always appear under the slash, no matter which way the slash is facing, thus: $\textsc{arg}\backslash\textsc{fn}$ and $\textsc{fn}/\textsc{arg}$.

Given a verb *loves* of category $(\textsc{dp}\backslash \mathrm{s})/\textsc{dp}$, we choose version (10a) of *everyone* and version (10d) of *no one*, and we have the following derivation of linear scope:

$$\dfrac{\text{everyone}_a{:}\mathrm{s}/(\textsc{dp}\backslash \mathrm{s}) \qquad \dfrac{\text{loves}{:}(\textsc{dp}\backslash \mathrm{s})/\textsc{dp} \quad \text{no one}_d{:}((\textsc{dp}\backslash \mathrm{s})/\textsc{dp})\backslash(\textsc{dp}\backslash \mathrm{s})}{\text{loves no one}_d{:}\textsc{dp}\backslash \mathrm{s}}<}{\text{everyone}_a\ (\text{loves no one}_d){:}\mathrm{s}}>$$

The < and > inferences are function application, with the arrow pointing in the direction of the argument. So the semantic value delivered by this derivation will be $\mathbf{everyone}_a(\mathbf{no\ one}_d\ \mathbf{loves}) = \forall x\neg\exists y.\mathbf{loves}\ y\ x$.

In order to arrive at inverse scope, Steedman provides **B** (“the Bluebird”, i.e., forward function composition), a combinator that allows composing the subject with the verb before combining with the direct object:

$$\dfrac{\dfrac{\text{everyone}_a{:}\text{s}/(\textsc{dp}\backslash\text{s}) \qquad \text{loves}{:}(\textsc{dp}\backslash\text{s})/\textsc{dp}}{\text{everyone}_a\ \text{loves}{:}\text{s}/\textsc{dp}}>\mathbf{B} \qquad\qquad \text{no one}_c{:}(\text{s}/\textsc{dp})\backslash\text{s}}{\text{everyone}_a\ \text{loves no one}_c{:}\text{s}}<$$

This derivation uses the same entry for *everyone* (namely, (10a)), but a different lexical entry for *no one*, (10c) instead of (10d). Semantically, the **B** inference corresponds to function composition: $\mathbf{no\ one}_c(\lambda x(\mathbf{everyone}_a(\mathbf{loves}\ x))) = \neg\exists y\forall x.\mathbf{loves}\ y\ x$.

Function composition is independently motivated by so-called non-constituent coordination, as in Right Node Raising examples such as *Ann described and Betty built the motorboat*: function composition allows treating the strings *Ann described* and *Betty built* as predicates with category s/dp. The conjunction of these constituents produces a conjoined function that applies to the right raised NP as an object, yielding a sentence.

Crucially, the order of syntactic combination differs across the two derivations just given: (*everyone* (*loves no one*)) for linear scope versus ((*everyone loves*) *no one*) for inverse scope. The claim, then, is that inverse scope is only possible if function composition has refactored the syntactic constituency, with concomitant changes in intonation and information structure.

Steedman (2012) develops the implications of this approach in depth, addressing many of the issues discussed in this article. In particular, he provides an independent mechanism for scoping indefinites involving Skolem functions. The behavior of indefinites, and the relevance of Skolem functions for describing that behavior, is the topic of section 5.

2.6 The logic of scope-taking

Lambek (1958) proposes using a substructural logic for modeling the syntax and the semantics of natural language. Developing Lambek’s approach, Moortgat (1988) offers an inference rule that characterizes scope-taking. He uses q to build the syntactic category of a scope-taking element. For instance, in Moortgat’s notation, *everyone* has category $q(\textsc{dp}, \text{s}, \text{s})$: something that functions locally as a dp, takes scope over an s, and produces as a result a (quantified) s.

$$\dfrac{\varDelta[A] \vdash B \qquad \varGamma[C] \vdash D}{\varGamma[\varDelta[q(A,B,C)]] \vdash E}\,q$$

This inference rule says that if $\varDelta$ is a syntactic structure in category B containing within it a constituent of category A, then if A is replaced by a scope-taking expression of category $q(A,B,C)$, the modified structure $\Delta[q(A,B,C)]$ can function in a larger derivation in the role of a C.

Although this inference rule says something deep and insightful about scope-taking, it is less than fully satisfying logically. For instance, there is no general corresponding right rule (rule of proof) that would fully characterize the logical content of scope-taking.

One notable feature of type-logical treatments is that the unary logical connectives $\diamond$ and $\Box^{\downarrow}$ provide a principled mechanism for managing scope islands. See Moortgat (1997) or Barker & Shan (2006) for details.

In addition to Moortgat's inference rule given above, there are at least three general type-logical approaches to scope. One strategy factors scope-taking into multiple logical modes that interact via structural postulates. Multimodal approaches include Morrill (1994); Moortgat (1995); Barker & Shan (2006); Barker (2007); Barker & Shan (2014).

Bernardi and Moortgat take a different tack, adapting an extension of Lambek grammar due to Grishin (1983) on which the multiplicative conjunction and its left and right implicative adjoints are dual to a cotensor, along with its adjoint operations. Moortgat (2009); Bernardi (2010); Bernardi & Moortgat (2010); Barker *et al.* (2010); Bastenhof (2013) explore the application of the Lambek-Grishin calculus to scope-taking in some detail.

Finally, Morrill *et al.* (2011) develop an extension of Lambek Grammar that allows syntactic structures to be discontinuous. Then a quantifier such as *everyone* can combine with the discontinuous constituent *John called __ yesterday* in order to form *John called everyone yesterday*.

Each of these approaches is discussed in more detail in Part II of Barker & Shan (2014).

3 Continuations, scope, and binding

Scope-taking occurs when an expression takes a portion of its surrounding context as its semantic argument. In the theory of programming languages (e.g., Wadler (1994)), the context of an expression is called its **continuation**. As might be expected, formal systems that explicitly manipulate continuations are well-suited to reasoning about scope-taking.

With hindsight, implicit use of continuations can be detected in a number of semantic theories. For instance, in the presentation of Hendriks' Flexible Montague Grammar above in section 2.4, the symbol 'κ' in the statement of Value Raising is precisely a variable over continuations. Other examples of theories that have a strong flavor of continuations, as discussed below, include Montague's conception of DP as a generalized quantifier, as well as the notion from dynamic semantics that a sentence denotes an update function on the rest of the discourse.

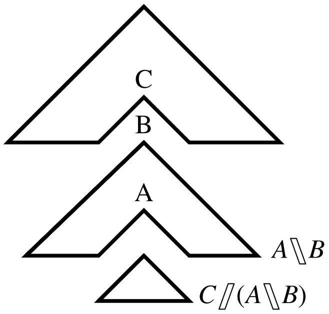
The first explicit use of continuations (and closely related techniques such as monads) to model natural language include Barker (2001, 2002); de Groote (2001); Shan (2001, 2005). The main applications of continuations in these analyses are scope-taking and binding. In this section, I will present a formal system developed in joint work with Chung-chieh Shan, as reported in Shan & Barker (2006) et seq. (see Barker & Shan (2014) for a comprehensive discussion). I will present this system in more detail than the theories surveyed in section 2. One payoff will be an account of the interaction of scope with binding on which weak crossover falls out from the nature of the basic scope-taking mechanism.

3.1 Syntactic categories for reasoning about scope-taking

Normally, functors combine with arguments that are syntactically adjacent to them, either to the left or the right. In the notation of categorial grammar (e.g., Lambek (1958)), a functor in category $A\backslash B$ combines with an argument to its left, and a functor in category B/A combines with an argument to its right. So if *John* has category DP, and *slept* has category DP\s, *John left* has category s.

For scope-taking, linear adjacency is not sufficient. After all, a scope-taker is not adjacent to its argument, it is contained within its argument. What we need is a syntactic notion of 'surrounding' and 'being surrounded by'. From a type-logical point of view, the needed categories are a second mode; see Barker & Shan (2006) or Part II of Barker & Shan (2014) for a development of the categories used here within the context of a substructural logic (i.e., a type-logical categorial grammar).

Pursuing this idea for now on a more informal, intuitive level, we will build up to a suitable category for a scope-taker in two steps. First, consider again the schematic picture of scope-taking:



The category of the notched triangle in the middle—the nuclear scope—will be $A\backslash\!\!\backslash B$: something that would be a complete expression of category B, except that it is missing an expression of category A somewhere inside of it. Just like $A\backslash B$, $A\backslash\!\!\backslash B$ will have semantic type $\mathtt{a} \to \mathtt{b}$: a function from objects of type $\mathtt{a}$ to objects of type $\mathtt{b}$, assuming that $\mathtt{a}$ and $\mathtt{b}$ are the semantic types of expressions in categories A and B.

Expressions in categories of the form $A\backslash\!\!\backslash B$ will play the role of continuations.

The second step is to consider the scope-taker itself. It takes the continuation above it as its semantic argument. But once again, it is not adjacent to its argument. Rather, it is surrounded by its argument. Just as we needed a notion of 'missing something somewhere inside of it', we now need a notion of 'missing something surrounding it'. If $A\backslash\!\!\backslash B$ means 'something that would be a B if we could add an A somewhere specific inside of it', then we'll use $C/\!\!/D$ to mean 'would be a C if there were a D surrounding it'. Of course these two notions complement each other; and in fact, a little thought will reveal that the surrounding D will always be a continuation. The general form of a scope-taker, then, will be $C/\!\!/(A\backslash\!\!\backslash B)$: something that combines with a continuation of category $A\backslash\!\!\backslash B$ surrounding it to form a result expression of category C.

For example, consider the sentence *John called everyone yesterday*. The nuclear scope is the sentence missing the scope-taker: *John called* [ ] *yesterday*. This is an expression that would be an s except that it is missing a DP somewhere inside of it. So this continuation has category $\textsc{dp}\backslash\!\!\backslash \mathrm{s}$. When the quantifier *everyone* combines with this continuation, it will form a complete sentence of category s. The syntactic category of the quantifier, then, will be $\mathrm{s}/\!\!/(\textsc{dp}\backslash\!\!\backslash \mathrm{s})$: the kind of expression that needs a continuation of category $\textsc{dp}\backslash\!\!\backslash \mathrm{s}$ surrounding it in order to form a complete s. The semantic type of *everyone* will be $(\mathtt{e} \to \mathtt{t}) \to \mathtt{t}$, just as expected for a generalized quantifier.

3.2 A continuation-based grammar

In a continuation-based grammar, every expression has access to (one of) its continuations. The challenge for a building such a grammar is figuring out how to combine two expressions, each of which expects to be given as its semantic argument a context containing the other. In order for this to work, the two expressions must take turns: one will play the role of context for the other, then vice versa. The question of which one serves as context first is precisely the question of what takes scope over what.

On the implementation level, the fragment as presented here takes the form of a combinatory categorial grammar, similar in many respects to those of Hendriks (1993); Jacobson (1999); Steedman (2001, 2012), in which a small number of type-shifters (“combinators”) adjust the syntactic categories and the meanings of constituents. It is a faithful both to the spirit and to many of the details of the formal fragment in Shan & Barker (2006). As mentioned above, a more extensive development can be found in Barker & Shan (2014).

The remainder of this subsection will set out the formal system in a way that is complete and precise, but rather dense. In the subsections that follow I will present the same system in ‘tower notation’, which is easier to grasp and use.

The scope-taking system relies on two type shifters, LIFT and LOWER. In these rules, the colon notation separates the semantic value of an expression from its syntactic category, so that $x : A$ stands for an expression having semantic value x with category A. Then for all semantic values x, and for all syntactic categories A and B,

$$\textsc{lift}(x{:}A) = (\lambda\kappa.\kappa x){:}B/\!\!/(A\backslash\!\!\backslash B)$$
$$\textsc{lower}(x{:}A/\!\!/(\textsc{s}\backslash\!\!\backslash\textsc{s})) = x(\lambda\kappa.\kappa){:}A$$

Type-shifters are allowed to apply to sub-categories in the following manner: if some type-shifter Σ is such that $\Sigma(x{:}A) \Rightarrow (fx){:}B$, then for all semantics values M and all syntactic categories C and D there is a related type-shifter Σ' such that $\Sigma'(M{:}C/\!\!/(A\backslash\!\!\backslash D)) \Rightarrow (Mf){:}C/\!\!/(B\backslash\!\!\backslash D)$. Although the application of type-shifters is sometimes constrained in the service of limiting overgeneration (e.g., Steedman (2001), Chapter 4), the combinators in the system presented here apply freely and without constraint.

In addition to the type-shifters, which operate on isolated expressions, there are three rules for combining expressions. For all semantic values x, y, f, M and N, and for all categories A, B, C, D, E, and F,

Forward combination: $f{:}B/A + x{:}A \Rightarrow ((\lambda xy.xy)\, f\, x){:}B$
Backward combination: $x{:}A + f{:}A\backslash B \Rightarrow ((\lambda xy.yx)\, f\, x){:}B$
Continuized combination: If $x{:}A + y{:}B \Rightarrow (fxy){:}C$, then

$$M{:}D/\!\!/(A\backslash\!\!\backslash E) + N{:}E/\!\!/(B\backslash\!\!\backslash F) \Rightarrow (\lambda\kappa.M(\lambda m.N(\lambda n.\kappa(fmn)))){:}D/\!\!/(C\backslash\!\!\backslash F)$$

Here, ‘+’ stands for the syntactic merge operation. The first two rules are the ordinary combination rules of categorial grammar. The third rule governs combination in the presence of scope-taking expressions. For instance, given that $\textsc{dp} + \textsc{dp}\backslash\textsc{s} \Rightarrow \textsc{s}$ (by backward combination), we have the following instance of continuized combination: $\textsc{s}/\!\!/(\textsc{dp}\backslash\!\!\backslash\textsc{s}) + \textsc{s}/\!\!/((\textsc{dp}\backslash\textsc{s})\backslash\!\!\backslash\textsc{s}) \Rightarrow \textsc{s}/\!\!/(\textsc{s}\backslash\!\!\backslash\textsc{s})$.

Recalling that we assigned the scope-taking expression *everyone* the syntactic category $\textsc{s}/\!\!/(\textsc{dp}\backslash\!\!\backslash\textsc{s})$, we have the following derivation for the sentence *everyone left*:

$$\textsc{lower}(\textsc{s}/\!\!/(\textsc{dp}\backslash\!\!\backslash\textsc{s})+\textsc{lift}(\textsc{dp}\backslash\textsc{s})) = \textsc{lower}(\textsc{s}/\!\!/(\textsc{dp}\backslash\!\!\backslash\textsc{s})+\textsc{s}/\!\!/((\textsc{dp}\backslash\textsc{s})\backslash\!\!\backslash\textsc{s})) \Rightarrow \textsc{lower}(\textsc{s}/\!\!/(\textsc{s}\backslash\!\!\backslash\textsc{s})) \Rightarrow \textsc{s}$$

with semantics

$$(\lambda\kappa.\mathbf{everyone}(\lambda m.(\lambda\kappa.\kappa(\mathbf{left}))(\lambda n.\kappa((\lambda xy.yx)\, m\, n))))(\lambda\kappa\kappa) \rightsquigarrow \mathbf{everyone}(\lambda m.\mathbf{left}\, m)$$

If we render the semantic value **everyone** as the generalized quantifier $\lambda\kappa\forall x.\kappa x$, the semantic value of the sentence reduces to $\forall x.\mathbf{left}x$.

As promised, the next subsections will provide an equivalent, somewhat more perspicuous presentation of the system.

3.3 Tower notation

In the tower notation, syntactic categories of the form $C/\!\!/(A\backslash\!\!\backslash B)$ can be written equivalently as $\dfrac{C\,|\,B}{A}$. So, in particular, the syntactic category for *everyone* is $\textsc{s}/\!\!/(\textsc{dp}\backslash\!\!\backslash\textsc{s}) \equiv \dfrac{\textsc{s}\,|\,\textsc{s}}{\textsc{dp}}$. Likewise, in the corresponding semantic values, $\lambda\kappa.g[\kappa f]$ can be written equivalently as $\dfrac{g[\;]}{f}$, so the denotation of *everyone* is $\lambda\kappa.\forall y.\kappa y \equiv \dfrac{\forall y.\,[\;]}{y}$.

The crux of the system is continuized combination:

(11)
$$\left(\begin{array}{cc} \dfrac{C\,|\,D}{A} & \dfrac{D\,|\,E}{A\backslash B} \\ \textit{left} & \textit{right} \\ \dfrac{g[\;]}{x} & \dfrac{h[\;]}{f} \end{array}\right) = \begin{array}{c} \dfrac{C\,|\,E}{B} \\ \textit{left right} \\ \dfrac{g[h[\;]]}{f(x)} \end{array}$$

On the syntactic level (the upper part of the diagram), the syntactic categories are divided into an upper part and a lower part by a horizontal line. Below the horizontal line is ordinary categorial combination, in this case, backward combination, i.e., $A + A\backslash B \Rightarrow B$. Above the horizontal line, the two inner category elements in $C|D + D|E$ cancel in order to produce $C|E$.

On the semantic level, below the horizontal line is normal function application: $f + x = f(x)$. Above the line is something resembling function composition: $g[\;] + h[\;] = g[h[\;]]$.

For example, here is a tower derivation of *everyone left*:

(12)
$$\left(\begin{array}{cc} \dfrac{\textsc{s}\,|\,\textsc{s}}{\textsc{dp}} & \dfrac{\textsc{s}\,|\,\textsc{s}}{\textsc{dp}\backslash\textsc{s}} \\ \textit{everyone} & \textit{left} \\ \dfrac{\forall y.\,[\;]}{y} & \dfrac{[\;]}{\mathbf{left}} \end{array}\right) = \begin{array}{c} \dfrac{\textsc{s}\,|\,\textsc{s}}{\textsc{s}} \\ \textit{everyone left} \\ \dfrac{\forall y.\,[\;]}{\mathbf{left}(y)} \end{array}$$

In this derivation, *left* has already undergone LIFTing. In tower, notation, the LIFT type-shifter looks like this (for all semantic values x and all syntactic categories A and B):

(13)
$$\boxed{\begin{array}{c} \\ A \\ \textit{phrase} \\ x \\ \\ \end{array} \;\overset{\text{LIFT}}{\Rightarrow}\; \begin{array}{c} \dfrac{B \mid B}{A} \\ \textit{phrase} \\ \dfrac{[\;]}{x} \end{array}}$$

This rule is a straightforward generalization of Partee's (1987) LIFT type-shifter.

(14)
$$\begin{array}{c} \mathrm{DP} \\ \textit{John} \\ \mathbf{j} \end{array} \;\overset{\text{LIFT}}{\Rightarrow}\; \begin{array}{c} \dfrac{\mathrm{S} \mid \mathrm{S}}{\mathrm{DP}} \\ \textit{John} \\ \dfrac{[\;]}{\mathbf{j}} \end{array} \qquad\qquad \begin{array}{c} \mathrm{DP}\backslash\mathrm{S} \\ \textit{left} \\ \mathbf{left} \end{array} \;\overset{\text{LIFT}}{\Rightarrow}\; \begin{array}{c} \dfrac{\mathrm{S} \mid \mathrm{S}}{\mathrm{DP}\backslash\mathrm{S}} \\ \textit{left} \\ \dfrac{[\;]}{\mathbf{left}} \end{array}$$

For instance, in (14a), LIFTing the proper name *John* into the quantifier category $\mathrm{S}/\!\!/(\mathrm{DP}\backslash\!\!\backslash\mathrm{S})$ yields the usual generalized quantifier semantics, namely $\lambda\kappa.\kappa\mathbf{j}$. Likewise, when *left* undergoes the LIFT typeshifter, the result in (14b) is the verb phrase that appears above in the derivation of *everyone left*. So on the continuations approach, Montague's conception of expressions in the category DP as uniformly denoting generalized quantifiers is simply a special case of a more general pattern, and follows directly from providing continuations systematically throughout the grammar.

Note that the final syntactic category of *everyone left* in (11) is $\dfrac{\mathrm{S} \mid \mathrm{S}}{\mathrm{S}}$ instead of a plain S. On the semantic level, converting back from tower notation to flat notation, the final denotation is $\lambda\kappa\forall y.\kappa(\mathbf{left}\,y)$.

This is the kind of meaning that characterizes a dynamic semantics. There are superficial differences: unlike the dynamic account of, for instance, Groenendijk & Stokhof (1991), the meaning here is not a relation between sets of assignment functions (in fact, the continuation-based system here is variable-free in the sense of Jacobson (1999), and does not make use of assignment functions at all). What makes this denotation a dynamic meaning is that it is a function on possible discourse continuations. In the terminology of dynamic semantics, a sentence meaning is a function for updating an ongoing discourse with the contribution of the local sentence. Thus the conception of a sentence meaning as a context update function follows as a special case of providing continuations systematically throughout the grammar.

Of course, if the sentence in (12) happens to be a complete discourse by itself, just as on any dynamic semantics, we need a way to close off further processing. We accomplish this with the LOWER type-shifter:

(15)
$$\boxed{\begin{array}{ccc} \dfrac{A\,|\,\mathrm{s}}{\textsc{s}} & \textsc{lower} & A \\ \textit{phrase} & \Rightarrow & \textit{phrase} \\ \dfrac{f[\;]}{x} & & f[x] \end{array}}$$

This type-shifter applies to the result above to yield the following truth value.

(16)
$$\begin{array}{ccc} \dfrac{\mathrm{s}\,|\,\mathrm{s}}{\textsc{s}} & \textsc{lower} & \textsc{s} \\ \textit{everyone left} & \Rightarrow & \textit{everyone left} \\ \dfrac{\forall y.\,[\;]}{\mathbf{left}\ y} & & \forall y.\,\mathbf{left}\ y \end{array}$$

The LOWER type-shifter plays a role that is directly analogous to Groenendijk & Stokhof's (1990) '↓' operator. Just like ↓, LOWER maps dynamic sentence meanings (in both cases, functions on surrounding discourse) into static propositions (in the extensional treatment here, truth values).

3.4 Directionality: explaining scope bias

There are two kinds of sensitivity to order that must be carefully distinguished here. The first kind is the directionality that is built into the categorial notation of the solid slashes. That is, an expression in category $A\backslash B$ combines with an argument to its left, and an expression in category B/A combines with an argument to its right. Nothing in the type-lifting system here disturbs this kind of directionality. For instance, the verb phrase *left* has category $\textsc{dp}\backslash\mathrm{s}$, and expects to find its subject to its left. After LIFTing, as shown in (11), it continues to expect its subject to its left.

The other kind of order sensitivity concerns scope-taking. This has to do with which expressions take scope over which other expressions. Crucially, there is a left-to-right bias built into the continuized combination rule. As a consequence of this bias, when a sentence contains two quantifiers, by default, the quantifier on the left takes scope over the one on the right:

(17)

$$\begin{array}{c}\dfrac{\text{S}\mid\text{S}}{\text{DP}}\\ \textit{someone}\\ \dfrac{\exists x.\,[\,]}{x}\end{array}\left(\begin{array}{c}\dfrac{\text{S}\mid\text{S}}{(\text{DP}\backslash\text{S})/\text{DP}}\\ \textit{loves}\\ \dfrac{[\,]}{\textbf{loves}}\end{array}\quad\begin{array}{c}\dfrac{\text{S}\mid\text{S}}{\text{DP}}\\ \textit{everyone}\\ \dfrac{\forall y.\,[\,]}{y}\end{array}\right)$$

$$= \begin{array}{c}\dfrac{\text{S}\mid\text{S}}{\text{S}}\\ \textit{Someone loves everyone}\\ \dfrac{\exists x.\,\forall y.\,[\,]}{\textbf{loves}\,y\,x}\end{array}\quad\overset{\text{LOWER}}{\Rightarrow}\quad\begin{array}{c}\text{S}\\ \textit{Someone loves everyone}\\ \exists x.\,\forall y.\,\textbf{loves}\,y\,x\end{array}$$

So on this approach, the bias towards linear scope is a result of the particular way in which the composition schema regulates the order of combination.

Now, the fact that the bias is left-to-right instead of right-to-left is a stipulation. It is possible to replace the rule as given with one on which the meaning of the expression on the right by default takes scope over (is evaluated before) the meaning of the expression on the left, given suitable corresponding adjustments in the syntactic portion of the combination rule (see Barker & Shan (2014), section 2.5 for details). So the *direction* of the bias does not follow from pursuing a continuation-based approach. What *does* follow is that a bias must be chosen, since there is no way to write down the continuized combination rule without making a decision about whether the expression on the left will by default take scope over the expression on the right, or vice versa. Unlike any of the strategies for scope-taking discussed above in section 2, then, the particular continuation-based strategy here forces explicit consideration of evaluation order, with consequences for default scope relations, and, as we see shortly, crossover effects.

3.5 Scope ambiguity

The left-to-right bias built into the combination scheme guarantees linear scope for any derivation that has a single layer of scope-taking, as we have seen. But of course sentences containing two quantifiers typically are ambiguous, having both a linear scope reading and an inverse scope reading. Clearly, then, inverse scope must require more than a single layer of scope-taking. This requires, in turn, generalizing type-shifters so that they can apply to a multi-story tower. We will accomplish this by allowing type-shifters to apply to subcategories, as spelled out above in section 3.2. In the tower notation, this amounts to requiring that whenever some type-shifter maps an expression of category A into category B, then the same type-shifter also maps any expression of category $\dfrac{\text{C}\mid\text{D}}{\text{A}}$ into category $\dfrac{\text{C}\mid\text{D}}{\text{B}}$.

In particular, for any category A, we have

$$\begin{array}{c} \mathrm{S}\,|\,\mathrm{S} \\ \hline \mathrm{DP} \\ \textit{everyone} \\ \forall x.[\;] \\ \hline x \end{array} \quad \overset{\mathrm{LIFT}}{\Rightarrow} \quad \begin{array}{c} \mathrm{S}\,|\,\mathrm{S} \\ \hline A\,|\,A \\ \hline \mathrm{DP} \\ \textit{everyone} \\ \forall x.[\;] \\ \hline [\;] \\ \hline x \end{array} \tag{1}$$

The semantics of this variation on the generalized LIFT interacts with the combination schema in such a way that within any given layer, quantifiers on the left still outscope quantifiers on the right, but any quantifier in a higher layer outscopes any quantifier on a lower layer. We can illustrate this with a derivation of inverse scope:

$$\begin{array}{c} \mathrm{S}\,|\,\mathrm{S} \\ \hline \mathrm{S}\,|\,\mathrm{S} \\ \hline \mathrm{DP} \\ \text{someone} \\ [\;] \\ \hline \exists x.\,[\;] \\ \hline x \end{array} \left(\begin{array}{c} \mathrm{S}\,|\,\mathrm{S} \\ \hline \mathrm{S}\,|\,\mathrm{S} \\ \hline (\mathrm{DP}\backslash \mathrm{S})/\mathrm{DP} \\ \text{loves} \\ [\;] \\ \hline [\;] \\ \hline \textbf{loves} \end{array} \quad \begin{array}{c} \mathrm{S}\,|\,\mathrm{S} \\ \hline \mathrm{S}\,|\,\mathrm{S} \\ \hline \mathrm{DP} \\ \text{everyone} \\ \forall y.\,[\;] \\ \hline [\;] \\ \hline y \end{array} \right) = \begin{array}{c} \mathrm{S}\,|\,\mathrm{S} \\ \hline \mathrm{S}\,|\,\mathrm{S} \\ \hline \mathrm{S} \\ \text{someone loves everyone} \\ \forall y.\,[\;] \\ \hline \exists x.\,[\;] \\ \hline \textbf{loves}\, y\, x \end{array}$$

$$\overset{\text{Lower}}{\Rightarrow} \quad \begin{array}{c} \mathrm{S}\,|\,\mathrm{S} \\ \hline \mathrm{S} \\ \text{someone loves everyone} \\ \forall y.[\;] \\ \hline \exists x.\,\textbf{loves}\, y\, x \end{array} \quad \overset{\text{Lower}}{\Rightarrow} \quad \begin{array}{c} \mathrm{S} \\ \text{someone loves everyone} \\ \forall y.\,\exists x.\,\textbf{loves}\, y\, x \end{array}$$

Because the internally-LIFTed version of *everyone* given in (16) allows the quantification introduced by the quantifier to take place on the top layer of the tower, it will outscope the existential introduced by *someone*, resulting in inverse scope, as desired.

3.6 Quantificational binding

In order to explain how the combination schema given above makes good predictions about weak crossover, it is necessary to give some details of how pronoun binding works in this system.

As in Jacobson (1999), the presence of an unbound pronoun will be recorded on the category of each larger expression that contains it. In particular, a clause containing an unbound pronoun will have category $\mathrm{DP} \rhd \mathrm{S}$ rather than plain S, with semantic type $\mathtt{e} \to \mathtt{t}$ (a function from individuals to sentence meanings). In order to accomplish this, pronouns will be treated as taking scope:

(18)

$$\left(\begin{array}{c}\dfrac{\text{DP} \rhd \text{S} \mid \text{S}}{\text{DP}} \\ \textit{he} \\ \dfrac{\lambda y.\,[\;]}{y}\end{array} \quad \begin{array}{c} \dfrac{\text{S}\mid\text{S}}{\text{DP}\backslash\text{S}} \\ \textit{left} \\ \dfrac{[\;]}{\textbf{left}}\end{array}\right) = \begin{array}{c}\dfrac{\text{DP}\rhd\text{S}\mid\text{S}}{\text{S}} \\ \textit{he left}\\ \dfrac{\lambda y.\,[\;]}{\textbf{left}\,y}\end{array} \overset{\text{LOWER}}{\Rightarrow} \begin{array}{c}\text{DP}\rhd\text{S}\\ \textit{he left}\\ \lambda y.\,\textbf{left}\,y\end{array}$$

The syntactic category of the pronoun is something that functions locally as a DP, takes scope over an S, and creates as a result an expression of category $\text{DP} \rhd \text{S}$.

If the category of a complete utterance is $\text{DP} \rhd \text{S}$, the value of the embedded pronoun must be supplied by the pragmatic context. But in the presence of a suitable quantifier, the pronoun can be bound. The binding variant of the quantifier *everyone* will have category $\dfrac{\text{S}\mid\text{DP}\rhd\text{S}}{\text{DP}}$ and semantics $\lambda\kappa\forall x.\kappa\,x\,x$: something that knows how to turn a sentence containing a pronoun ($\text{DP} \rhd \text{S}$) into a plain clause by semantically duplicating an individual and using the second copy to provide the value of the pronoun.[1]

We immediately have an account of quantificational binding:

(19)

$$\left(\begin{array}{c}\dfrac{\text{S}\mid\text{DP}\rhd\text{S}}{\text{DP}} \\ \textit{everyone's} \\ \dfrac{\forall x.[\;]x}{x}\end{array} \quad \begin{array}{c}\dfrac{\text{DP}\rhd\text{S}\mid\text{DP}\rhd\text{S}}{\text{DP}\backslash\text{DP}} \\ \textit{mother} \\ \dfrac{[\;]}{\textbf{mom}}\end{array}\right)\left(\begin{array}{c}\dfrac{\text{DP}\rhd\text{S}\mid\text{DP}\rhd\text{S}}{(\text{DP}\backslash\text{S})/\text{DP}} \\ \textit{loves} \\ \dfrac{[\;]}{\textbf{loves}}\end{array} \quad \begin{array}{c}\dfrac{\text{DP}\rhd\text{S}\mid\text{S}}{\text{DP}} \\ \textit{him} \\ \dfrac{\lambda y.\,[\;]}{y}\end{array}\right)$$

$$= \begin{array}{c}\dfrac{\text{S}\mid\text{S}}{\text{S}} \\ \textit{Everyone's mother loves him} \\ \dfrac{\forall x.(\lambda y.\,[\;])x}{\textbf{loves}\;x\;(\textbf{mom}\;y)}\end{array} \overset{\text{LOWER}}{\Rightarrow} \begin{array}{c}\text{S} \\ \textit{Everyone's mother loves him} \\ \forall x.(\lambda y.\textbf{loves}\;x\;(\textbf{mom}\;y))x\end{array}$$

After beta reduction, the semantic value is $\forall x.\,\textbf{loves}\,(\textbf{mom}\;x)\;x$.

Note that the quantifier has no difficulty scoping out of the possessor phrase (this required an application of Value Raising in Flexible Montague Grammar).

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[1] We can derive the binding version of any DP via a type-shifting rule, if desired; see Barker & Shan (2014), chapter 2.

3.7 C-command is not required for quantificational binding

In order for a universal quantifier to bind a pronoun, it is necessary for the quantifier to at least take scope over the pronoun. Most theories of binding (e.g., Büring (2004)) require further that the quantifier c-command the pronoun (simplifying somewhat, from the surface syntactic position of the quantifier). But as the derivation in (19) shows, the universal has no difficulty binding the pronoun in the system here despite the fact that it does not c-command the pronoun.

In fact, the standard wisdom notwithstanding, the facts do not support requiring quantifiers to c-command the pronouns they bind:

(20) a. [Everyone$_i$'s mother] thinks he$_i$'s a genius.
b. [Someone from every$_i$ city] hates it$_i$.
c. John gave [to each$_i$ participant] a framed picture of her$_i$ mother.
d. We [will sell no$_i$ wine] before it$_i$s time.
e. [After unthreading each$_i$ screw], but before removing it$_i$...
f. The grade [that each$_i$ student receives] is recorded in his$_i$ file.

This data shows that quantifiers can bind pronouns even when the quantifier is embedded in a possessive DP, in a nominal complement, in a prepositional phrase, in a verb phrase, in a temporal adjunct, even when embedded inside of a relative clause. In each example, the quantifier does not c-command the pronoun. Barker (2012) argues that although various modifications and extensions of c-command have been proposed to handle some of the data, none of these redefinitions covers all of the data.

As the derivation in (19) shows, it is perfectly feasible to build a grammar in which a quantifier can bind a pronoun without c-commanding it. Nothing special needs to be said; indeed, we would need to take special pains to impose a c-command requirement.

Denying that c-command is required for binding is not the same as saying that a quantifier can bind any pronoun that follows it. If the quantifier is embedded in a scope island, it cannot bind a pronoun outside of that island.

(21) a. Someone who is from every city$_i$ loves it$_{*i}$.
b. Someone from every city$_i$ loves it$_i$.

Relative clauses are particularly strong scope islands. A binding relationship between the quantifier and the pronoun in (21a) is impossible not because the quantifier fails to c-command the pronoun, but because the quantifier is embedded in a relative clause. As (21b) shows, when the quantifier is no longer inside a relative clause, binding becomes possible, despite the fact that the quantifier still does not c-command the pronoun.

3.8 Crossover

Continuations are particularly well-suited for reasoning about order of evaluation. For instance, in the theory of computer programming languages, Plotkin (1975) explores call-by-name versus call-by-value evaluation disciplines by providing a continuation-passing style transform. As emphasized in Shan & Barker (2006), the continuation-based approach allows a principled strategy for managing evaluation order in natural language.

In the application of order of evaluation to crossover, we note that a quantifier must be evaluated before any pronoun that it binds. As discussed above, this requirement is built into the composition schema given above. To see this, consider what happens when a pronoun precedes a potential quantificational binder in a simple example:

(22)

$$\left(\begin{array}{cc} \frac{\mathrm{DP} \rhd \mathrm{S} \mid \mathrm{S}}{\mathrm{DP}} & \frac{\mathrm{S} \mid \mathrm{S}}{\mathrm{DP}\backslash\mathrm{DP}} \\ \text{his} & \text{mother} \end{array}\right)\left(\begin{array}{cc} \frac{\mathrm{S} \mid \mathrm{S}}{(\mathrm{DP}\backslash\mathrm{S})/\mathrm{DP}} & \frac{\mathrm{S} \mid \mathrm{DP} \rhd \mathrm{S}}{\mathrm{DP}} \\ \text{loves} & \text{everyone} \end{array}\right) = \begin{array}{c} \frac{\mathrm{DP} \rhd \mathrm{S} \mid \mathrm{DP} \rhd \mathrm{S}}{\mathrm{S}} \\ \text{his mother loves everyone} \end{array}$$

The prediction is that this string will be ungrammatical on an intended reading on which which the quantifier binds the pronoun. Combination proceeds smoothly, and the complete string is recognized as a syntactic (and semantic) constituent; but the result is not part of a complete derivation of a clause. In particular, the final result can't be lowered, since the category of the expression does not match the input to the LOWER type-shifter, which requires a category of the form $\frac{A \mid \mathrm{S}}{\mathrm{S}}$. This means that at the end of the derivation, the pronoun continues to need a binder, and the quantifier continues to need something to bind.

It is important to emphasize that the evaluation-order constraint is not simply a linear order restriction. This is crucial, since there are well-known systematic classes of examples in which a quantificational binder linearly follows a pronoun that it nevertheless binds. Reconstruction provides one such class of cases:

(23) a. Which of his$_i$ relatives does everyone$_i$ love the most?
b. the relative of his$_i$ that everyone$_i$ loves the most

A complete explanation of these reconstruction cases would require a discussion of wh-movement, pied-piping, and relative clause formation. But once these independently-motivated elements are in place, the binding analyses of the sentences in (23) follow automatically, without any adjustment to the lexical entries of the quantifier, of the pronoun, any of the type shifters defined above, and without modifying the combination schema. (See Shan & Barker (2006); Barker (2009, 2014); Barker & Shan (2014) for details.)

In sum, we have seen how a continuation-based grammar can provide an account of scope-taking on which providing continuations systematically throughout the grammar unifies Montague's conception of DP's as generalized quantifiers with the dynamic view of sentence meaning as context update as two special cases of a general strategy: the first follows from continuizing the category DP, and the second follows from continuizing the category S.

Furthermore, we have seen how the general linear scope bias, as well as basic weak crossover examples, falls out from a requirement for left-to-right evaluation. In general, then, one of the distinctive advantages of continuations is that they provide a principled framework for reasoning about order effects related to scope-taking. In addition to crossover and reconstruction, evaluation order has empirical consequences for the interaction of scope with superiority, negative polarity licensing, discourse anaphora, and donkey anaphora. These phenomena will not be discussed in detail here in this short article, but they are all discussed in depth in Barker & Shan (2014).

4 Kinds of scope-taking

In the canonical cases of scope-taking—the only kind discussed so far—the situation is relatively simple: the scope-taking expression is a single constituent, the nuclear scope surrounds the scope-taker, the root of the nuclear scope dominates every part of the scope-taker, no part of the scope-taker dominates any part of the nuclear scope. This section discusses a variety of other kinds of scope-taking, including lowering, split scope, existential versus distributive scope, parasitic scope, and recursive scope. Discussion of the various techniques that are specific to managing the scope-taking of indefinites (including 'pseudoscope') is postponed to section 5 below.

4.1 Lowering ('total reconstruction')

Since May (1977):188 there have been suggestions that in some highly restricted circumstances, some quantifiers can take scope in a position that is lower than their surface position:

(24) a. Some politician$_i$ is likely [t_i to address John's constituency].
  b. There is a politician x such that x is likely to address John's constituency.
  c. The following is likely: that there is a politician
    who will address John's constituency.

On the assumption that *some politician* is related to the subject position of the infinitival verb *to address* via movement from the position marked t_i, the two interpretations of (24a) given in (24b) and (24c) can be explained by supposing that *some politician* moves downward into the lower position, where it is able to take scope over only the bracketed embedded clause. This is sometimes known as **total reconstruction** (see Sauerland & Elbourne (2002)). Keshet (2010) gives an analysis that does not involve downward movement.

4.2 Split scope

Jacobs (1980) suggests that the German determiner *kein* 'no', contributes two semantic elements that take scope independently of one another. More specifically, he proposes that the semantics of *kein* involves negation and existential quantification, and that other scope-takers could intervene between the negation and the existential (see Geurts (1996) and de Swart (2000) for discussion of the pros and cons of a split-scope analysis of German *kein*).

Similarly, Cresti (1995):99, following Higginbotham (1993) (see also Ginzburg & Sag (2000) for an alternative analysis) suggests that some wh-phrases, including *how many* questions, contribute two scope-taking elements, namely, a wh-operator over numbers (*what number n*) and a generalized quantifier (*n-many people*):

(25) a. How many people should I talk to?
  b. What number n is such that there are n-many people I should talk to?
  c. What number n is such that I should talk to n-many people?

The first reading asks how many people have the property of my needing to talk to them. The second reading asks for a number such that it is necessary for me to talk to that many people. The difference between the readings depends on whether the generalized quantifier element of the split meaning takes scope above or below *should*.

Heim (2001) and Hackl (2000) argue for a split-scope analysis for comparatives and superlatives (see also discussion in Szabolcsi (2010):168).

(26) a. This paper is 10 pages long. It is required to be exactly 5 pages longer than that.
b. required > (d = 15) > a d-long paper: it is necessary for the paper to be exactly 15 pages long.
c. (d = 15) > required > a d-long paper: the maximum length such that the paper is required to be at least that long is 15 pages.

The ambiguity is analyzed by assuming that the comparative operator *–er* takes split scope. The reading in (26b) arises when *required* takes scope over both parts contributed by *–er*, and the reading in (26c) arises when the top part of the split scope of *–er* takes wider scope over *required*.

In terms of the categories for scope-taking introduced in section 3, split scope corresponds to a category for the scope-taking expression in which the local syntactic category it itself scope-taking. That is, given an ordinary scope-taking category schema such as $\frac{E \mid F}{A}$, we can instantiate A as a category that is itself the category of a scope-taking expression, e.g., $\frac{E \mid F}{\left(\frac{C \mid D}{B}\right)}$. In QR terms, one way of thinking of this kind of situation is that instead of leaving behind a simple trace (say, an individual-denoting variable), the scope-taking expression leaves behind a denotation with a higher type which is itself capable of taking scope.

4.3 Existential versus distributive quantification

Szabolcsi (e.g., Szabolcsi (2010) Chapter 7) argues that many quantifiers exhibit a systematic kind of split scope. One of the scope-taking elements gives rise to existential quantification, the other, something she calls ‘distributive’ quantification (roughly, universal quantification). She motivates this claim with an example from Ruys (1993), discussed by Reinhart (1997) and many others, involving an indefinite containing a plural NP:

(27) a. If three relatives of mine die, I’ll inherit a house.
b. If there exists any set of three relatives who die, I’ll inherit a house.
c. There exists a set of three relatives each with the following property: if that person dies, I’ll inherit a house.
d. There exists a set of three relatives such that if each member of that set dies, I’ll inherit a house.

There is an irrelevant narrow-scope reading of the indefinite given in (27b), which says that if any set of three relatives die, I’ll inherit a house. The reading of interest is the one on which there is a specific set of three relatives, perhaps the ones who have a prior claim on the inheritance, and the speaker will inherit the house only if all of them are out of the way. The puzzle is that if the indefinite takes wide scope with respect to the conditional, then on most theories of scope, the identity of the house will depend on the choice of the relative, and we expect there to be as many as three inherited houses, as in the paraphrase given in (27c). But the strongly preferred reading, perhaps the only wide-scope reading, is the one paraphrased in (27d), on which there need be no more than one house. In Szabolcsi's terminology, the existential scope of the indefinite can escape from the conditional, but the distributive scope—evoked informally here by the *each* in the paraphrase—remains clause-bounded, and trapped inside the antecedent. (See section 5 below for a discussion of the scope of indefinites.)

Universal quantifiers arguably also exhibit both existential and distributive scope.

(28) Every child tasted every apple. [Kuroda (1982)]

There is an ambiguity in (28) depending on whether the children all tasted apples from a jointly held set of apples, or whether each child tasted from a distinct set of apples specific to that child. We can understand this ambiguity as depending on whether the existential scope of the universal *every apple* is narrower or wider than the distributive scope of the higher universal *every child*.

On the categorial characterization of split scope above, a schematic category for *everyone* might be $\begin{array}{c} \exists X.[\;] \\ \hline \forall x \in X.[\;] \\ \hline x \end{array} : \begin{array}{c} \mathrm{s}\,|\,\mathrm{s} \\ \hline \mathrm{s}\,|\,\mathrm{s} \\ \hline \mathrm{DP} \end{array}$. Here, the upper existential expresses the existential scope of the quantifier, and the universal quantifier in the middle layer expresses its distributive scope. Note that on this lexical entry, given the tower system explained in section 3, the existential scope will always be at least as wide as the distributive scope.

The interaction of scope with distributivity is an intricate topic; see Szabolcsi (2010) Chapter 8.

4.4 Parasitic scope

In parasitic scope (Barker (2007)), one scope-taker takes scope in between some other scope-taker and that second scope-taker's nuclear scope. As a result, parasitic scope cannot occur without there being at least two scope-taking elements involved. The main application for parasitic scope in Barker (2007) involves 'sentence-internal' readings of *same* and *different*. The sentence-internal reading of *everyone read the same book*, for instance, asserts the existence of a book such that every person read that book.

The idea of parasitic scope can be illustrated with QR-style logical forms.

1. everyone[read[the[same book]]]
2. everyone($\lambda x.[x$[read[the[same book]]]])
3. everyone(same($\lambda f \lambda x.[x$[read[the[f(book)]]]]))

In step (1), both scope-taking elements are in their original surface syntactic positions. In step (2), *everyone* takes (covert) scope over the entire rest of the sentence, as per normal. In step (3), *same* takes scope. However, it does not take scope over the entire sentence, but only over the nuclear scope of *everyone*. Because this can only happen if *everyone* has already taken scope, the scope-taking of *same* is parasitic on the scope-taking of *everyone*.

In terms of the categories developed in section 3, the category of parasitic *same* is $\frac{\textsc{dp}\backslash\!\!\backslash\textsc{s} \,|\, \textsc{dp}\backslash\!\!\backslash\textsc{s}}{\mathrm{ADJ}}$. In order to unpack this category, recall that the category of *everyone* is $\textsc{s}/\!\!/(\textsc{dp}\backslash\!\!\backslash\textsc{s})$. In particular, the category of *everyone*'s nuclear scope is $\textsc{dp}\backslash\!\!\backslash\textsc{s}$. So the category for *same* is suitable for an expression that functions locally as an adjective, and takes scope over an expression of category $\textsc{dp}\backslash\!\!\backslash\textsc{s}$—that is, it takes scope over the nuclear scope of *everyone*.

Parasitic scope has been used to characterize a number of different phenomena. Kennedy & Stanley (2009) propose a parasitic scope analysis for sentences like *The average American has 2.3 kids*, resolving the puzzle posed by the fact that no individual person can have a fractional number of kids.

1. [[the[average American]][has[2.3 kids]]]
2. 2.3(λd.[[the[average American]][has[d-many kids]]])
3. 2.3(average($\lambda f \lambda d$.[[the[f(American)]][has[d-many kids]]]))

In step (2), the cardinal *2.3* takes scope, creating the right circumstance for *average* to take parasitic scope. Kennedy and Stanley provide details of the denotation for the *average* operator that gives suitable truth conditions for this analysis.

Parasitic scope allows for bound pronouns to be analyzed as scope-takers. The idea that anaphors might take scope is discussed by Dowty (2007). Morrill *et al.* (2011) give an account in their Discontinuous Lambek Grammar in terms of constituents with two discontinuities. The analysis can be translated into parasitic scope by assigning a bound pronoun such as *he* category $\frac{\textsc{dp}\backslash\!\!\backslash\textsc{s} \,|\, \textsc{dp}\backslash\!\!\backslash\textsc{s}}{\mathrm{DP}}$:

1. everyone[said[he left]]
2. everyone(λx.[x [said[he left]]])
3. everyone(he($\lambda y \lambda x$.[x [said[y left]]]))

If the denotation of the pronoun is $\lambda\kappa\lambda x.\kappa x x$, then each individual chosen by the universal will be duplicated, then fed to the parasitic nuclear scope twice, simultaneously controlling the value of x and of y.

Parasitic scope analyses have also been proposed for various types of coordination in English and in Japanese (Kubota & Levine (2012); Kubota (2013)).

4.5 Recursive scope

Yet another logical possibility is for a scope-taking element to produce a result category that is itself scope-taking. Schematically, this would be a category of the form $\dfrac{\left(\dfrac{\mathrm{D}\,|\,\mathrm{E}}{\mathrm{C}}\right)\Bigg|\,\mathrm{B}}{\mathrm{A}}$. This is the category of an expression that functions locally as an expression in category A, that takes scope over a containing expression of category B, and turns that surrounding expression into something in the result category $\frac{\mathrm{D} \mid \mathrm{E}}{\mathrm{C}}$. But since this result category is itself a scope-taking category, the result after the first scope-taking is an expression that still needs to take (even wider) scope. This is the idea of recursive scope.

Solomon (2010) argues that recursive scope is required to analyze internal readings of *same* in the presence of partitivity.

(29) Ann and Bill know [some of the same people].

On the simple parasitic analysis of *same* described above in the previous subsection, the truth conditions predicted there require that there is some set of people X such that Ann and Bill each know a subset of X. But nothing in that analysis prevents the subsets from being disjoint, so that there might be no one that Ann and Bill both know, contrary to intuitions about the meaning of (29).

Instead, Solomon suggests that the category of *same* should be $\frac{\left(\frac{\mathrm{DP}\backslash\!\!\backslash \mathrm{S} \,\mid\, \mathrm{DP}\backslash\!\!\backslash \mathrm{S}}{\mathrm{DP}}\right) \,\Big|\, \mathrm{DP}}{\mathrm{A}}$. On this analysis, *same* first takes scope over the DP *some of the __ people*; it then turns this DP into a parasitic scope-taker that distributes over the set containing Ann and Bill.

On the recursive-scope analysis proposed by Solomon, then, *same* is an operator that turns its nuclear scope into a new, larger scope-taking expression.

For a second example of a recursive scope analysis in the literature, Barker (2013); Barker & Shan (2014) argues that in Andrews Amalgams such as *Sally ate* [*I don't know what __*] *today*, the bracketed clause functions as a DP. Crucially, the interpretation of the elided wh-complement (__) takes the continuation of the bracketed expression as its antecedent. This can be analyzed as the sluice gap taking scope over the bracketed clause, and turning it into a continuation-consuming (i.e., scope-taking) generalized quantifier.

5 Indefinites

The scope behavior of indefinites has inspired considerable theoretical creativity.

Dynamic semantics, one of the main semantic approaches in recent decades, was developed in large part to reconcile the scope behavior of indefinites with their binding behavior. A discussion of dynamic semantics appears in section 6 below.

This section discusses indefinites as referential expressions or as singleton indefinites; Skolem functions and choice functions, branching quantifiers, the Donald Duck problem, cumulative readings, and the de dicto/de re ambiguity. See Ruys (2006) and Szabolcsi (2010) for additional discussion.

5.1 Referential indefinites vs. wide-scope indefinites

In the earliest accounts, including May (1977), indefinites were treated as existential quantifiers, and so participated in Quantifier Raising just like other quantifiers. The hope was that all scope taking would behave in a uniform way, and in particular with respect to scope islands. The fact that the scope of universals is for the most part clause bounded (see section 1.6 above) led to the expectation that the scope of indefinites would be too.

But the scope of indefinites is not clause bounded.

(30) Nobody believes the rumor that a (certain) student of mine was expelled.

Fodor & Sag (1982) noted that (30) has a reading on which the speaker may have a specific student in mind, as if the indefinite took scope over the entire sentence, despite its being embedded inside of a clausal nominal complement (a particularly strong scope island for universal quantifiers).

Fodor and Sag suggested that in addition to the usual quantificational meaning, indefinites can have a specific or referential interpretation. Schwarzschild (2002) proposes a similar but distinct idea by noting that pragmatic domain restriction can narrow the set of objects in the extension of the indefinite's nominal to a single entity, what he calls a **singleton indefinite**. He argues that *certain* signals that the indefinite is quantifying over a singleton domain. Singleton indefinites behave logically as if they were referential or scopeless.

Complicating the picture, an indefinite can take wide scope with respect to scope islands at the same time that it takes narrow scope with respect to some other operator in the sentence (Farkas (1981 [2003]); Abusch (1993)).

(31) a. Each student read every paper that discussed some problem.
b. Every student is such that there is some problem such that
the student read every paper that discussed the problem.

Farkas observes that sentences like (31a) have a reading on which the indefinite *some problem* takes scope over *every paper*, yet does not take scope over *each student*, so that each student studied a different problem.

As another example of a class of quantifiers whose scope-taking constraints differ from those of distributive universals, Carlson (1977) observed that bare plurals typically take the narrowest possible scope.

5.2 Skolemization

The challenges of accounting for wide-scope indefinites motivate a number of analyses that rely on higher-order quantification and Skolem functions.

Skolem (1920[1967]) proved that it is always possible to replace existential quantifiers with operations over the set of individuals that are (now) called **Skolem functions**. For instance, the formula $\forall x \exists y.Px \wedge Qy$ is true iff $\forall x.Px \wedge Q(fx)$ is satisfiable, where f is a variable over Skolem functions with type $\mathtt{e} \to \mathtt{e}$.

In order to simulate an existential in the scope of more than one universal, the Skolem function must take as arguments variables controlled by each of the universals that outscope it. Thus $\forall w \forall x \exists y \forall z.R(\,w,\ x,\ y,\ z)$ is equivalent to $\exists f \forall w \forall x \forall z.R(\,w,\ x,\ f(w,x),\ z)$, where f is a function of type $\mathtt{e} \to \mathtt{e} \to \mathtt{e}$. The fact that f is sensitive to the choice of w and of x, but not of z, encodes the fact that the existential in the original formula is within the scope of the first two universals, but not of the third.

The original application of Skolemization has to do with proof theory. In its applications in natural language semantics, Skolemization provides a highly expressive way to characterize scope dependencies, as the next subsection shows.

5.3 Branching quantifiers

What happens when an existentially-quantified variable is replaced with a Skolem function that ignores some of the universals that outscope it? The result can express truth conditions that are not equivalent to any linear scoping of first-order universals and existentials. These **branching quantifiers** can be thought of as a partially-ordered set of quantifiers. For example, Hintikka (1974) offers a branching-quantifier analysis of the following sentence:

(32) Some relative of each villager and some relative of each townsman hate each other.

$$\begin{pmatrix} \forall x\ \exists x' \\ \forall y\ \exists y' \end{pmatrix}.(\textbf{villager}\ x \wedge \textbf{townsman}\ y) \to (\textbf{rel}\ x\, x' \wedge \textbf{rel}\ y\, y' \wedge \textbf{hate}\ x' y')$$

The idea is that the choice of x' depends on the choice of x in the usual way, and likewise, the choice of y' depends on the choice of y; but the choice of x' does not depend on the choice of y or y', nor does the choice of y' depend on the choice of x or x'. The intended interpretation can be made precise with Skolem functions:

$$\exists f \exists g \forall x \forall y.(\textbf{villager}\ x \wedge \textbf{townsman}\ y) \to (\textbf{rel}\ x\,(fx) \wedge \textbf{rel}\ y\,(gy) \wedge \textbf{hate}\,(fx)(gy))$$

where f and g are variables over functions with type $\mathtt{e} \to \mathtt{e}$. Crucially, the identity of $f(x)$ depends only on f and on x, but not on y, and symmetrically for $g(y)$. That means that f allows us to choose a villager's relative without regard to which townsman we have in mind. The Skolemized formula therefore requires that the selected villager must hate the full set of townsman relatives in the range of g.

There is no way for these truth conditions to be accurately expressed by a linear scoping of the quantifiers. For example, the linear scoping

$$\forall x \forall y \exists x' \exists y' [(\textbf{villager}\ x \wedge \textbf{townsman}\ y) \to (\textbf{rel}\ x\, x' \wedge \textbf{rel}\ y\, y' \wedge \textbf{hate}\ x' y')]$$

allows us to switch to a different townsman relative for each choice of a villager relative; on the branching reading just characterized, we have to stick with a single choice of one relative per villager or townsman.

There is some doubt that natural language expresses genuine branching quantifiers. See Westerståhl (this volume), Fauconnier (1975), Barwise (1979), Sher (1990), Beghelli *et al.* (1997), Szabolcsi (1997), and Szabolcsi (2010):209 for discussions of branching quantifiers in natural language. Schlenker (2006) argues that there are branching quantifiers after all; but before discussing his argument below in section 5.6, it is first necessary to bring choice functions into the picture.

5.4 Motivating choice functions: the Donald Duck problem

Any complete theory of scope-taking must explain how the scope of indefinites escapes from islands. Reinhart (1997) points out that there is one way to handle wide-scope indefinites that is clearly wrong: leaving the descriptive content in place, but allowing (only) the existential quantifier to take arbitrarily wide scope.

(33) a. If we invite a certain philosopher to the party, Max will be annoyed.
b. There is some entity x such that if x is a philosopher
and we invite x to the party, Max will be annoyed.

Moving just the existential to the front of the sentence gives rise to the paraphrase in (33b). But the truth conditions in (33b) are too weak for any natural interpretation of (33a), since they are verified by the existence of any entity that is not a philosopher. For instance, the fact that Donald Duck is not a philosopher makes (33b) true.

Reinhart (1992, 1997); Winter (1997, 2004), and many others suggest that the Donald Duck problem and other considerations motivate representing indefinites using choice functions. (See also Egli & Von Heusinger (1995) for a separate proposal to use choice functions to interpret indefinites.) A choice function maps a property to an object that satisfies that property. If P is a property of type $\mathtt{e} \rightarrow \mathtt{t}$, then any choice function f will have type $(\mathtt{e} \rightarrow \mathtt{t}) \rightarrow \mathtt{e}$, and will obey the following rule: $P(fP)$, that is, $f($**woman**$)$ must choose an individual who has the property of being a woman. Special care must be taken to deal with the possibility that the property P might be empty.

Quantifying over choice functions solves the Donald Duck problem, since we can now give the following analysis for (33a):

(34) a. $\exists f$.if we invite f(philosopher), Max will be annoyed.
b. There is some choice function f such that if we invite the philosopher
chosen by f to the party, Max will be annoyed.

Instead of quantifying over individuals, we quantify over choice functions. Then the truth conditions will require that there be some way of choosing a philosopher such that if we invite that particular philosopher, Max will be annoyed. We achieve the effect of choosing a philosopher before executing the conditional, but without moving any lexical material out of the conditional.

5.5 Pseudoscope

Kratzer (1998) proposes an analysis similar to that depicted in (34a), but without explicit quantification over choice functions:

(35) If we invite f(philosopher), Max will be annoyed.

Here, the choice function f is a free variable whose value must be supplied by context. Presumably the speaker has in mind some way of selecting a particular philosopher.

On this view, the appearance that the indefinite is taking wide scope is just an illusion arising from the contribution that contextually-supplied choice functions make. It's not really wide scope, it's **pseudoscope**. And if what looks like wide scope is really pseudoscope, this clears the way to assuming that all true scope-taking uniformly obeys scope islands.

There is a lively debate over whether it is descriptively adequate to leave choice functions unquantified. Chierchia (2001) and others argue that negation and other downward-monotonic operators require explicit quantification over choice functions. See Szabolcsi (2010) Section 7.1 for a summary of the debate so far.

5.6 Skolemized choice functions

Based on the data we've seen so far, we could consider simply exempting indefinites from scope islands. Allowing indefinites to take extra wide scope (e.g., through QR) always gives reasonable results (i.e., leads to interpretations that are intuitively available). However, there appear to be cases in which a simple no-island strategy undergenerates.

In general, we can consider Skolemized choice functions, which take zero or more individuals plus one property as arguments, returning an individual that possesses that property: type $\texttt{e} \rightarrow ... \rightarrow \texttt{e} \rightarrow (\texttt{e} \rightarrow \texttt{t}) \rightarrow \texttt{e}$, where the number of initial individual-type arguments can be as few as zero.

Building on observations of Chierchia (2001) and Geurts (2000) and others, Schlenker (2006) argues that indefinites can be functionally dependent on other quantifiers in a way that motivates Skolemized choice functions.

(36) a. If every student improves in a (certain) area, no one will fail the exam.

b. $\exists f.(\forall x.\textbf{student}\ x \rightarrow \textbf{improves-in}(f\ x\ \textbf{area})\ x) \rightarrow \neg\textbf{fail}$

Here, f is a Skomenized choice function with type $\texttt{e} \rightarrow (\texttt{e} \rightarrow \texttt{t}) \rightarrow \texttt{e}$. For at least some speakers, (36) has a reading on which it existentially quantifies over functions from students to areas. These truth conditions cannot be rendered by first-order quantifiers (given normal assumptions about the meaning of the conditional): giving the existential wide scope over the universal is too restrictive, since it requires there to be a single area that all the students improve in. Giving the existential narrow scope under the universal is too permissive, since the sentence will be true just in case each student improves in any area, even if it's not their weakest area.

Schwarz (2001, 2011) points out that unconstrained Skolemized choice functions are not available with *no*:

(37) No student read a book I had recommended.
$\exists f \neg \exists x.\mathbf{student}\ x \wedge \mathbf{read}(f\ x\ \mathbf{recommend})\ x$

By selecting a perverse choice for f, the truth conditions as given can be verified even if each student read a book I had recommended, contrary to intuitions.

If the described reading of (36) is indeed a legitimate interpretation of the sentence in question, Skolemized choice functions, or something equivalent to them, are necessary for a complete description of scope in natural language.

5.7 Cumulative readings

There is another type of reading often attributed to sentences involving cardinal quantifiers that cannot be expressed by linear scope relations:

(38) a. Two boys read three books.
b. two > three: Two boys are such that each of them read three books
c. three > two: Three books are such that each of them was read by two boys
d. cumulative: a group of two boys were involved in reading a set of three books.

On the subject-wide-scope interpretation, reading three books is a property that at least two boys have. On the object-wide-scope reading, being read by two boys is a property that at least three books have. On the reading of interest here, there is a group of at least two boys whose net amount of book-reading sums to at least three books. This is called a 'cumulative' or a 'scopeless' reading. If we allow that quantifiers can have both existential and universal scope (as discussed in section 4.3), we can suppose that the existential scope of each cardinal is wider than both of their universal scopes. This would have the effect of holding the set of boys and the set of books constant. Questions would remain concerning how the scopes of the universals correspond to the participation of the individuals in the described event (must each boy read some of each book?). In any case, neither of the traditional scope interpretations, as paraphrased in (38b) and (38c), gives the desired reading. See Westerståhl (this volume), Szabolcsi (2010) Chapter 8, or Champollion (2010) for guides to the literature on cumulativity.

5.8 De dicto/de re

There can be variability as to which person's beliefs support the applicability of descriptive content. This variability is often assumed to be a scope ambiguity:

(39) a. Lars wants to marry a Norwegian.
b. $\mathbf{wants}(\exists x.\mathbf{norwegian}\ x \wedge \mathbf{marry}\ x\ \mathbf{lars})\ \mathbf{lars}$
c. $\exists x.\mathbf{norwegian}\ x \wedge \mathbf{wants}(\mathbf{marry}\ x\ \mathbf{lars})\ \mathbf{lars}$

The sentence in (39a) can be used to describe a situation in which Lars has a desire that the person he marries will be from Norway, or else a situation in which there is someone Lars wants to marry, and that person happens to be Norwegian. If we imagine that the indefinite might take scope either within the embedded clause, as in (39b), or else at the level of the matrix clause, as in (39c), we get something roughly in line with these two interpretations. In (39b), the property of being a Norwegian is part of the desire report, but in (39c), it is outside of the desire report. The scoping in (39c) guarantees the existence of a specific person in the real world, and is called **de re** (‘of the thing’), in contrast with the scoping in (39b), which is **de dicto** (‘of the word’).

There are many puzzle cases in which simple scope relations do not appear to give a complete picture of the facts.

(40) Mary wants to buy an inexpensive coat.

For instance, Fodor (1970); Szabó (2010) observes that in addition to the standard de dicto reading (Mary wants to save money) and the standard de re reading (she’s picked out a coat, but doesn’t know its inexpensive), (40) can be used to describe a situation in which Mary has narrowed down her choices to a small set of coats without picking a specific one, so the truth conditions of giving the indefinite wide scope aren’t satisfied; and yet she isn’t aware that the coats are inexpensive, so the truth conditions of giving the indefinite the narrow scope aren’t satisfied.

Reconciling these and other examples with a scope-based approach requires making a number of extra assumptions. See Keshet (2010) for a proposal.

6 Dynamic semantics

File Change Semantics (Heim (1982)) and Discourse Representation Theory (Kamp (1981); Kamp & Reyle (1993)) address the specialness of indefinites by supposing that indefinites add a novel discourse referent to the discourse representation. Dynamic Predicate Logic ('DPL', Groenendijk & Stokhof (1991)) and Dynamic Montague Grammar ('DMG', Groenendijk & Stokhof (1990)) implement a similar idea, taking inspiration from Dynamic Logic (e.g., Harel (1984)), a formal system designed for reasoning about the semantics of computer programming languages. In DPL, sentences denote relations over assignment functions. Adopting the notation of Muskens (1996), A_x *man entered* translates as $[x|\textbf{man}\,x, \textbf{entered}\,x]$, where $[x_n|\text{test}_1, \text{test}_2, ...]$ is defined to be

$$\{< i, j > |i \text{ and } j \text{ differ at most in what they assign to } x_n, \text{ and } j \in \text{test}_1 \wedge j \in \text{test}_2, ...\}.$$

The heart of the matter is the way in which conjunction works from left to right:

$$[\![A \textit{ and } B]\!] = \{\langle i, k\rangle | \exists j : \langle i, j\rangle \in [\![A]\!] \wedge \langle j, k\rangle \in [\![B]\!]\}$$

That is, the interpretation of the coordination of A followed by B proceeds left to right: first, associate the input assignments i with each of their updated output assignments j reflecting the content of A; then take the intermediate assignments j as the input to B.

To see how this works, let a sequence of objects such as "acb" represent the partial assignment function g such that $g(x) = a$, $g(y) = c$, and $g(z) = b$.

$$\begin{bmatrix} \text{abc} \\ \text{acb} \end{bmatrix} \begin{matrix} \text{a}_y \text{ man entered} \\ \rightarrow \end{matrix} \begin{bmatrix} \text{aac} \\ \text{adc} \\ \text{aec} \\ \text{aab} \\ \text{adb} \\ \text{aeb} \end{bmatrix} \begin{matrix} \text{he}_y \text{ sat down} \\ \rightarrow \end{matrix} \begin{bmatrix} \text{aac} \\ \text{adc} \\ \text{aab} \\ \text{adb} \end{bmatrix}$$

Note that sequences of sentences are treated as if they had been conjoined. The indefinite in the first sentence introduces a range of candidates for the value of its index, and the pronoun in the second sentence refers back to that index. In more detail, the update effect of A_y *man entered* will be to relate each assignment function in the input set to a set of all assignments that are as similar as possible except perhaps that the second position (corresponding to the variable y associated with the use of the indefinite) contains a man who entered. (In this model, apparently, the men who entered are a, d, and e.) The update effect of he_y *sat down* will be to eliminate those assignments in which the second position contains a man who did not sit down. The net effect is that the set of output assignments will have all and only men who entered and sat down in their second column.

Although this system deals with the existential effect of an indefinite, as well as the persistence of the binding effect of an indefinite, it has nothing new to say about scope-taking. In fact, in order to handle displaced scope and scope ambiguity, these systems must be supplemented with a theory of scope-taking (e.g., Quantifier Raising). The relevance of dynamic approaches for a theory of scope is that that they allow a treatment of certain binding phenomena that might have seemed inconsistent with independent constraints on scope-taking, as in donkey anaphora:

(41) a. Every man [who owns a$_x$ donkey] beats it$_x$.
b. If [a man owns a$_x$ donkey], he beats it$_x$.

Under normal assumptions (widely adopted, though challenged in Barker & Shan (2008)), we certainly don't want the indefinite to take scope over either the universal in (41a) or the conditional in (41b). That would entail the existence of one special donkey, which is not the reading of interest. The puzzle is that if the scope of the indefinite is trapped inside the bracketed clauses, how does it come to bind a pronoun outside of its scope domain?

On the dynamic approach, the indefinites can take scope within the bracketed expressions, and yet still provide discourse referents for the pronoun to refer to, in the same way (as we have seen) that the indefinite in the sentence *A$_y$ man entered* can provide a discourse referent for a pronoun in a subsequent sentence such as *He$_y$ sat down* without needing to take scope over the second sentence.

7 Hamblin Semantics

We have seen in the discussion of dynamic semantics in the previous section that there is a deep connection between existential quantification and tracking multiple alternatives. The formal systems mentioned in section 6 tracked alternatives by providing a distinct assignment function for each alternative. However, similar strategies are possible that involve tracking other types of denotations.

Following Kratzer & Shimoyama (2002), one such strategy is known as **Hamblin semantics**. Hamblin (1973) proposes that questions denote a set of propositions, where each proposition provides an answer to the question. In Hamblin semantics as applied to indefinites, the usual meanings are replaced with sets of meanings, where each element in the set corresponds to a different way of resolving the value of an indefinite.

Because predicates and arguments now denote sets of functors and sets of objects, function application must be generalized to apply 'pointwise' in the following manner. If $[A/B + B]$ is a function/argument construction in which the pre-Hamblinized types are `b` → `a` and `b`, then in a Hamblin setting, the types will be lifted into sets: (`b` → `a`) → `t` and `b` → `t` Then Hamblin pointwise function application for sets of denotations will be as follows:

(42) $[\![A/B + B]\!] = \{fb \mid f \in [\![A/B]\!], b \in [\![B]\!]\}$

There is some discussion about the best way to generalize other semantic operations to a Hamblin setting, in particular, Predicate Abstraction (see Shan (2004); Novel & Romero (2010)).

Most expressions will denote the singleton set containing their pre-Hamblinized denotation; for instance, if the pre-Hamblinized verb *left* denotes the function **left** of type `e` → `t`, the Hamblinized version will denote the singleton set {**left**}.

Then indefinites simply denote the set consisting of all of the possible values that satisfy the restriction of the indefinite. For example, if **a**, **b**, and **c** are the women, then the denotation of *a women* will be {**a**, **b**, **c**}, and the composition of this set with the Hamblinized *left* will be {**left a**, **left b**, **left c**}. A sentence will be considered true just in case at least one of the propositions in the set denoted by the sentence is true.

Because pointwise composition allows the indeterminacy introduced by the indefinite to expand upwards throughout the composition in a potentially unbounded way, Hamblin semantics can simulate wide scope for indefinites independently of the action of QR (or of any other scope-taking mechanism). An example will show how this works:

1. a woman: {**a**, **b**, **c**}
2. saw (a woman): {**saw a**, **saw b**, **saw c**}
3. everyone (saw (a woman)): {**e'one**(**saw a**), **e'one**(**saw b**), **e'one**(**saw c**)}

Here, the Hamblinized denotation of *everyone* is the singleton set containing the usual generalized quantifier. Since the sentence will be true just in case at least one of the three alternatives is true, and since each alternative guarantees the existence of a single woman seen by everyone, the Hamblin treatment of this sentence is equivalent to the reading on which *a woman* receives wide scope.

One distinctive property of Hamblin systems is that the indefinite introduces indeterminacy, but the quantificational force of the alternative set depends on operators higher in the composition. This allows treatments of phenomena such as free choice *any* (and free choice permission, for Hamblin treatments of disjunction) on which the higher operator is construed as conjunction rather than as disjunction. (See, e.g., Kratzer & Shimoyama (2002) or Alonso-Ovalle (2006).)

Because indefinites in effect take scope via an independent mechanism, Hamblinization allows indefinites to take scope independently of other quantifiers. For instance, if we implemented tensed clauses as scope islands in a Quantifier Storage system by requiring that the quantifier store be empty before an embedded clause can combine with an embedding predicate, an indefinite inside the embedded clause could still take scope wider than the embedded clause, since placing restrictions on the quantifier store would not affect the set of alternatives used to encode the nondeterminism introduced by the indefinite.

In order for *Everyone saw someone* to receive linear scope, there must be a (Hamblinized, i.e., alternative-aware) existential operator that takes narrower scope than the universal.

On the natural assumption that disjunction introduces alternatives in a way that is similar to indefinites (Alonso-Ovalle (2006)), the Hamblin approach makes it natural to assume that disjunction has scope properties similar to indefinites. See Partee & Rooth (1983); Larson (1985); Hendriks (1993); Den Dikken (2006); Schlenker (2006) for discussions of the scope-taking of disjunction.

8 Computational processing

Managing ambiguity is a major challenge for natural language processing. The number of distinct legitimate scope interpretations for a sentence can be factorial in the number of scope-taking elements. For the same reason that it would be computationally inefficient to compute or store two distinct interpretations for a sentence containing an ambiguous word such as *bat* or *bank*, it would be inefficient to compute or store every disambiguated scope interpretation. Therefore computational linguists have devised schemes for representing meanings that are **underspecified** for scope, that is, neutral across scopings.

Cooper storage (discussed above in section 2.3) can serve to illustrate the basic idea. Consider a simple sentence containing multiple quantificational DPs immediately before the quantifiers have been removed from the store. The sentence is fully parsed, and all grammatical uncertainty has been resolved except for which quantifier will outscope the other. In this situation, the sentence with its unordered quantifier store constitutes a representation that is underspecified for scope.

Several underspecification strategies have been proposed that place constraints on logical representations, including Hole Semantics (Bos (2001)) and Minimal Recursion Semantics (Copestake *et al.* (2005)). The constraints for *someone loves everyone* would include requiring that *everyone* take scope over a sentence, that it bind a trace in the object position of *loves*, and so on. One of the main challenges in this research area is to find a constraint system such that finding one or finding all of the fully-specified representations is tractable.

See Fox & Lappin (2006) or the papers in Koller & Niehren (1999) for recent discussion.

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