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Association with distributivity and the problem of multiple antecedents for singular *different*

Chris Barker and Dylan Bumford, *NYU Linguistics*

Brasoveanu 2011 argues that certain expressions exhibit what he calls "association with distributivity" (AWD for short):

(1) a. Every boy read a different poem.
    b. The boys read a different poem.

The claim is that in (1a), *different* can be anaphorically linked to the distributivity introduced by the quantificational determiner *every*, in which case (1a) entails that no poem was read by more than one boy. In contrast, in (1b) the plural *the boys* does not introduce distributivity, which is why (on the AWD account) (1b) does not have the reading just described. Instead, (1b) only has an external reading on which *different* is anaphoric to some element outside to the sentence.

But the implementation of the AWD strategy in Brasoveanu 2011 systematically undergenerates possible interpretations:

(2) a. Every boy claimed that every girl read a different poem.
    b. Every boy made the following claim: that no two girls read the same poem.
    c. For every boy x there is a different poem y such that x claimed every girl read y

For instance, the Brasoveanu 2011 fragment predicts that (2a) has only one internal reading, the one paraphrased in (2b). The reason is that the formal analysis forces *different* to associate with whichever distributive operator takes narrowest scope. But native speakers report that (2a) can also have a paraphrase as in (2c), on which *different* associates with the wider-scope distributive operator.

We show one concrete way to extend the association-with-distributivity approach to handle the ambiguity illustrated in (2). Although the extension provides a reasonable account of the data, it makes the computational burden imposed by the AWD approach dramatically worse: on the implementation in Brasoveanu 2011, association with distributivity requires at least doubling the amount of contextual information that must be tracked by the compositional semantics; but under the extended implementation here, the amount of information that must be tracked is exponential in the number of distributive operators (though as in Brasoveanu's fragment, this additional information only exists in the scope of the distributor).

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1. AWD in de Groote's continuation-based dynamic semantics

We will develop an analysis here that we intend to faithfully embody the spirit of the AWD approach, but which differs from the implementation in Brasoveanu 2011 technically in many ways, both small and large. In addition to providing an AWD account of the ambiguity in (2), having a second implementation of association with distributivity will allow us to discuss below what is essential to the approach, and what is implementation-specific.

Association with distributivity is named on analogy with association with focus. In association with focus, focus creates a two-part structured meaning consisting of a focused denotation and a background. This two-part meaning is propagated upwards, where it can be accessed by a focus-sensitive operator such as *only*. Association with distributivity works in the opposite direction, from top to bottom: a distributive operator (such as *every*) creates a two-part context, which is passed downwards to subconstituents, where it can be accessed by a distributivity-sensitive operator such as *different*.

The backbone of the compositional analysis here will be the continuation-based dynamic semantics of de Groote 2007, except that instead of evaluating expressions with respect to a single sequence of individuals as in de Groote, here we will provide a pair of sequences, which will serve as the two-part context required by the AWD strategy.

Types		Variables	Examples
Entity:		x, y	**a**, **b**, **c**, ...
Stack:	list of Entities	i, j	**ab**, **aac**, ...
Bool:	truth value		TRUE, FALSE
Dref:	discourse referent (integer)	n, m, l	0, 1, 2, ...
Predicate:	Dref → Proposition	P, Q	**entered**, **sat**, ...
Continuation:	Stack → Stack → Bool	κ	$\lambda i j.\text{TRUE}$
Proposition:	Stack → Stack → Continuation → Bool	p, q	

There are three base types: entities, truth values, and discourse referents. Each discourse referents is implemented as an integer, interpreted as a position in a sequence of individuals. We will follow Brasoveanu 2011 in calling a sequence of individuals a 'stack', and we will use the following notation:

Notation	Gloss	Example
i_n	the nth element in the stack i	$(\mathbf{abc})_0 = \mathbf{a}$
$x{:}i$	the list formed by prepending x to i	$\mathbf{a}{:}(\mathbf{abc}) = \mathbf{aabc}$
$i^{x/n}$	the list formed by inserting x in position n of i	$(\mathbf{abc})^{\mathbf{a}/2} = \mathbf{abac}$

Dynamic propositions are modeled here as update functions that need three things in order to deliver a truth value: two independent stacks, representing the context in which the proposition is evaluated, and a continuation, representing the rest of the discourse. A simple example will show the role the continuation plays in dynamic interpretation (see de Groote 2007 for motivation and additional discussion):

(3) a. John entered and he$_0$ sat.
b. John: $\lambda Pij\kappa.P0i^{\mathbf{a}/0}j^{\mathbf{a}/0}\kappa$
c. entered: $\lambda nij\kappa.i_n, j_n \in \{\mathbf{a},\mathbf{b}\} \wedge \kappa ij$
d. and: $\lambda pq.p;q$, where $p;q \equiv \lambda ij\kappa.pij(\lambda i'j'.qi'j'\kappa)$
e. he$_n$: $\lambda Pij\kappa.Pnij\kappa$
f. sat: $\lambda nij\kappa.i_n, j_n \in \{\mathbf{a}\} \wedge \kappa ij$

The essentially dynamic element here is the conjunction in (3d), which evaluates the second conjunct (q) with respect to context stacks that have been updated by evaluation of the left conjunct (p).

In somewhat more detail: the proper name *John* (functioning as a generalized quantifier) pushes the individual John, represented here by the object **a**, onto both of the stacks in its context, and instructs the verb phrase (P) to look for its subject in the 0th position of the updated stacks. The verb phrase *entered* checks to make sure that the objects in the designated position of the stacks both have the property of entering, then passes the (previously updated) contexts that survive this test on to its continuation (i.e., the rest of the sentence). In this case, the continuation is the right conjunct, *he*$_0$ *sat*. The pronoun does not add any new item to the stacks, but tells its verb phrase where to look for its subject, in this case, once again, the 0th position. The net effect guarantees that as long as the index of the pronoun is 0, the entity that sat in this discourse will be the same entity that entered. More precisely, the discourse 'and (john entered)(he$_0$ sat)' will evaluate to TRUE when applied to any initial pair of stacks and the trivial continuation $\textsc{triv} = \lambda ij.\textsc{true}$.

The central idea of the AWD strategy is that distributive operators like *every* manipulate the two context stacks in a coordinated way:

(4) a. Every^0 boy entered.
b. every^n: $\lambda PQij\kappa.(\forall x, y, x \neq y : Pni^{x/n}j^{y/n}\textsc{triv} \rightarrow (Pn;Qn)i^{x/n}j^{y/n}\textsc{triv}) \wedge \kappa ij$,
c. boy: $\lambda nij\kappa.i_n, j_n \in \{\mathbf{a},\mathbf{b}\} \wedge \kappa ij$

As in Brasoveanu 2011, distributivity requires dual quantification, that is, quantification over distinct pairs of individuals. (Assume this *every* presupposes there are at least two distinct objects in the extension of its restrictor.) For every choice of distinct x and y in the domain, we update i and j by inserting x in the nth position of i, and y in the nth position of j. Then every choice of distinct x and y that satisfies the restrictor (i.e., every choice of x and y such that $Pni^{x/n}j^{y/n}\textsc{triv} = \textsc{true}$) must also satisfy the (dynamic conjunction of the restriction and the) nuclear scope, namely, $(Pn;Qn)i^{x/n}j^{y/n}\textsc{triv}$. If this test is passed, the original context is passed on to the global continuation without updating.[2] Then (4a) will evaluate to true just in case the set of boys is a subset of the set of people who entered.

[2]The fact that the continuation sees only the original, unupdated context means that, in the terminology of Groenendijk and Stokhof 1991, this *every* is externally dynamically closed. We can implement the quantificational subordination analysis of Brasoveanu 2011:130 by just moving the continuation inside the scope of the quantification, i.e., eliminate the final conjunct (κij) and replace the second occurrence of TRIV with κ.

The indefinite article will non-deterministically provide a potentially distinct object for each context stack:

(5) a. A^0 boy sat.
b. a^n: $\lambda PQij\kappa.\exists x, y : (Pn; Qn)i^{x/n} j^{y/n}\kappa$
c. sat: $\lambda nij\kappa.i_n, j_n \in \{\mathbf{a}\} \wedge \kappa ij$

After update with (5a), the context will contain stacks whose first elements are boys who sat.

Universal and existential quantification interact in a reasonable way:

(6) a. Every^0 boy recited a^1 poem.
b. every^0 boy ($\lambda n.$ a^1 poem ($\lambda m.$ recited m n))
c. recited: $\lambda mnij\kappa.\langle i_n, i_m\rangle, \langle j_n, j_m\rangle \in \{\langle \mathbf{a}, \mathbf{c}\rangle, \langle \mathbf{b}, \mathbf{d}\rangle\} \wedge \kappa ij$
d. poem: $\lambda nij\kappa.i_n, j_n \in \{\mathbf{c,d}\} \wedge \kappa ij$

After scoping the two generalized quantifiers, the logical form for (6a) will be as in (6b). As long as there is some way of choosing poems for every distinct pair of boys such that each boy read the corresponding poem, the sentence will evaluate as true.

With the distributivity machinery set up, adding a lexical entry for singular *different* is fairly elegant:

(7) a. Every^0 boy recited a^1 different poem.
b. every^0 boy ($\lambda n.$ a^1 (different poem) ($\lambda m.$ recited m n))
c. different: $\lambda Pm.Pm; (\lambda ij\kappa.i_m \neq j_m \wedge \kappa ij)$

Here, since boy **a** recited poem **c** and boy **b** recited poem **d**, (7a) will evaluate to true.[3]

(8) a. Every^0 boy enjoyed a^1 poem.
b. Every^0 boy enjoyed a^1 different poem.
c. enjoyed: $\lambda mnij\kappa.\langle i_n, i_m\rangle, \langle j_n, j_m\rangle \in \{\langle \mathbf{a}, \mathbf{c}\rangle, \langle \mathbf{b}, \mathbf{c}\rangle\} \wedge \kappa ij$

But if both boys enjoyed only poem **c**, then (8a) is true while (8b) is false.

In Brasoveanu 2011, context components are not stacks, as here, but sets of stacks, what Brasoveanu calls 'information states'. Thus where we have a pair of stacks, Brasoveanu would have a pair of sets of stacks. In other work (e.g., Brasoveanu 2007), Brasoveanu shows how dealing in information states rather than stacks can provide an account of strong

[3]In Brasoveanu 2011, *different* has two indicies: one that matches the index on the indefinite determiner it occurs immediately under, and another giving an offset value to use for finding the object to use for comparison. Neither of these indices are necessary here: the index of the indefinite is already available as the first argument to the nominal modified by *different*; and because this implementation does not make use of Brasoveanu's concatenation operator, the object to use for comparison will simply be found in the same column of the second context stack. (In the generalized fragment given in the next section, we will need to add a parameter distinct from any of the ones in Brasoveanu's account in order to disambiguate the antecedent of *different* for examples like (2a).)

versus weak readings of donkey anaphora; however, the reconstruction of association-with-distributivity here shows that the full power of information states is unnecessary to account for the sentence-internal reading of *different*, since stacks are enough.

2. An ambiguity: singular *different* can choose among multiple distributive antecedents

The fragment in Brasoveanu 2011 allows only one distributive operator at a time to control the extra information channel. Furthermore, the distributivity operator that is in control must always be the most local one (local in terms of narrowest scope). This is too restrictive, as the following examples show:

(9) a. Every boy gave every girl a different poem.
b. Every boy gave every girl he liked a different poem.
c. Every boy said every girl read a different poem.
d. Every boy said every girl read a different poem from a different book.

In the Brasoveanu 2011 fragment, (9a) will have two distinct readings, depending on whether *every boy* takes wide scope, in which case no boy gives the same poem to multiple girls, or whether *every girl* does, in which case no girl receives the same poem from multiple boys. But native speakers report that (9b) has the same ambiguity, even though the pronoun in *every girl he liked* forces the subject DP to take wide scope in order to bind the pronoun. Furthermore, (9c) appears to have the same ambiguity, even though the scope of *every* DPs are generally trapped inside of tensed clauses. Finally, despite the fact that (9d) can have only one scoping of the universal quantifiers, it can have four distinct interpretations, depending on which distributive operator each of the *different*s associates with.

The reason that Brasoveanu's implementation of the AWD strategy predicts only a single reading (per scoping) is because it allows for only a single extra information channel. So if there are two distributivity operators, in order for the operator with narrower scope to take control over the context, it must discard the information placed there by the first distributivity operator. Thus the distributivity operator with the narrowest scope will always be the only available antecedent for singular *different*.

The scope of indefinites is relevant here. We can arrive at an account of (9a) if we assume that indefinites can take wide scope:

(10) (every boy) λx ((a different poem) λz ((every girl) λy [x gave y z])).

As long as *different* is not in the scope of *every girl*, there is no obstacle to taking the distributivity introduced by *every boy* as antecedent, as desired.

It is unlikely that this strategy will work in general, however:

(11) Every$_1$ photographer claimed that [each$_2$ woman] preferred [a different$_1$ picture of herself$_2$].

In order for the anaphor *herself* to be bound by *each woman*, it must be in the scope of the quantifier. Yet it remains possible for *different* to take the higher quantifier as its antecedent. The relevant reading is one that would be entailed if each photographer claimed that each woman preferred the photograph of herself that that photographer had taken. Note that reconstruction is not relevant here: even if it were possible to reconstruct a portion of a raised indefinite, the truth conditions would entail that each photograph was a photograph of every woman, which is not required on the relevant reading.

We can use bound pronouns to impose a specific scoping relation over all three quantifiers:

(12) a. Each_1 traffic engineer insisted that [every_2 intersection she_1 controlled] ought to have [a different_1 speed at which its_2 lights changed].
b. Each_1 professor asked [every_2 student in her_1 class] to present [a different_1 paper by his_2 favorite author].

Assuming bound readings as indicated, in (12a) *each traffic engineer* must take scope over the DP *every intersection she controlled*, and the indefinite *a different speed at which its lights changed* must take scope narrower than the DP introduced by *every*. Then since (12a) has an interpretation that entails that no two traffic engineers insist on the same speed, it must be possible for *different* to be in the scope of both distributive operators and yet still take the higher operator as its antecedent.

A second argument along similar lines comes from inverse linking:

(13) a. Each polling company interviewed [a different person from every city].
b. Every unrelated language has [a different morpheme for marking each case].

It is generally accepted (see, e.g., the discussion in Heim and Kratzer 1998:233) that in inverse linking cases, quantifiers external to the inverse-linked DP cannot intervene in scope between the indefinite and the universal. That is, on any reading of the sentences in (13) in which the embedded universals take scope over the indefinite that contains them, the subject universal cannot take scope between the embedded universal and the indefinite. Thus since (13a) has a reading on which it entails that no two polling companies interviewed the same person, it must be possible for *different* to take *each* as its controlling distributor despite also being in the scope of *every*.

In view of these arguments, we will assume that it is possible for singular *different* to take a non-local distributive operator as antecedent.[4]

In order to generalize the fragment to handle this kind of ambiguity, instead of a pair of context stacks, we need in a list of stacks, a list whose length is unbounded.

[4]This means, incidentally, that although *different* can often be legitimately used to emphasize a wide-scope reading for an indefinite, as in, e.g., Breuning 2010:292, the presence of *different* does not guarantee wide scope for the indefinite that contains it.

Types		Variables	Examples
Stacklist:	list of Stacks	c	[**ab**], [**ab**, **aac**], ...
Continuation:	Stacklist → Bool	κ	TRIV $\equiv \lambda c.$TRUE
Proposition:	Stacklist → Continuation → Bool	p, q	

This change will simplify the types of continutions and propositions as shown. There will be a number of related changes throughout the fragment. In particular, we must generalize universal and existential quantification to handle lists of stacks:

(14) a. $p; q \equiv \lambda c\kappa.pc(\lambda c'.qc'\kappa)$

b. every^n: $\lambda PQc\kappa.(\forall x, y, x \neq y : Pnc'\text{TRIV} \rightarrow (Pn; Qn)c'\text{TRIV}) \wedge \kappa c$,
where length$(c) = m$ and $c' = [c_0^{x/n}, c_0^{y/n}, c_1^{x/n}, c_1^{y/n}, ..., c_m^{x/n}, c_m^{y/n}]$

c. boy: $\lambda nc\kappa.(\forall i \in c : i_n \in \{\mathbf{a}, \mathbf{b}\}) \wedge \kappa c$

d. gave: $\lambda mnlc\kappa.(\forall i \in c : \langle i_l, i_m, i_n\rangle \in \{\langle \mathbf{a}, \mathbf{e}, \mathbf{c}\rangle, \langle \mathbf{b}, \mathbf{f}, \mathbf{d}\rangle\}) \wedge \kappa c$

e. girl: $\lambda nc\kappa.(\forall i \in c : i_n \in \{\mathbf{e}, \mathbf{f}\}) \wedge \kappa c$

f. a^n: $\lambda PQc\kappa.\exists x_1, x_2, ..., x_m : (Pn; Qn)c'\kappa$,
where length$(c) = m$ and $c' = [c_0^{x_1/n}, c_1^{x_2/n}, ..., c_m^{x_m/n}]$

g. different^n: $\lambda Pm.Pm; (\lambda c\kappa.(c_0)_m \neq (c_n)_m \wedge \kappa c)$

h. poem: $\lambda nc\kappa.(\forall i \in c : i_n \in \{\mathbf{c,d}\}) \wedge \kappa c$

In the generalized universal in (14a), for every choice of distinct x and y in the domain, we replace each stack in c with a pair of stacks identical to c, except that one has x inserted in nth position, while the other has y inserted in the same position. So if the original stacklist $c = [\mathbf{abc}, \mathbf{dbc}]$, then the stacklist c' modified by *every*^{0} would be $[\mathbf{xabc}, \mathbf{ydbc}]$. The generalized existential in (14e) quantifies over each stack in the stacklist independently. As in the two-stack fragment above, this allows us to compare drefs across elements in the distributor. Note that both of the quantifier definitions in the earlier fragment are special cases of the ones here. The output contexts of the two-stack and multi-stack universals will be the same whenever the input context to the multi-stack universal contains one copy of the information in the two-stack input. For example, in the multi-stack fragment we have $every^0 PQ[\mathbf{abc}]\kappa = (\forall x, y, x \neq y : Pn[\mathbf{xabc}, \mathbf{yabc}]\text{TRIV} \rightarrow (Pn; Qn)[\mathbf{xabc}, \mathbf{yabc}]\text{TRIV}) \wedge \kappa[\mathbf{abc}]$. In the single-stack fragment we have $every^0 PQ[\mathbf{abc}, \mathbf{abc}]\kappa = (\forall x, y, x \neq y : Pn[\mathbf{xabc}, \mathbf{yabc}]\text{TRIV} \rightarrow (Pn; Qn)[\mathbf{xabc}, \mathbf{yabc}]\text{TRIV}) \wedge \kappa[\mathbf{abc}, \mathbf{abc}]$. The multi-stack existential reduces to the two-stack existential any time the input context contains exactly two stacks.

The additional readings for (9a) are now accounted for:

(15) a. (every^0 boy) (λl. (every^1 girl) (λn. (a^2 (different^1 poem)) (λm. gave $n\ m\ l$)))

b. (every^0 boy) (λl. (every^1 girl) (λn. (a^2 (different^2 poem)) (λm. gave $n\ m\ l$)))

The only difference is the choice of stack parameter indicated as a superscript on *different*. The goal is to choose the stack in the context list that differs from the first one only in its choice of the relevant distributed variable. Consider (9a). If the input context to this sentence is [**ab**], then the first universal duplicates this stack and adds a single point of variation (two distinct boys) in the 0th column. The second universal induces another mitotic duplication, this time with variation (two distinct girls) introduced in the 1st column. Finally, the existential injects a non-deterministic column of poems, one in the 2nd slot of each of the four stacks. These updates are schematically represented below:

$$\begin{bmatrix}\mathbf{a} & \mathbf{b}\end{bmatrix} \xrightarrow{\textit{every}^0\ \textit{boy}} \begin{bmatrix}\mathbf{x} & \mathbf{a} & \mathbf{b} \\ \mathbf{y} & \mathbf{a} & \mathbf{b}\end{bmatrix} \xrightarrow{\textit{every}^1\ \textit{girl}} \begin{bmatrix}\mathbf{x} & \mathbf{u} & \mathbf{a} & \mathbf{b} \\ \mathbf{x} & \mathbf{v} & \mathbf{a} & \mathbf{b} \\ \mathbf{y} & \mathbf{u} & \mathbf{a} & \mathbf{b} \\ \mathbf{y} & \mathbf{v} & \mathbf{a} & \mathbf{b}\end{bmatrix} \xrightarrow{a^2\ \textit{poem}} \begin{bmatrix}\mathbf{x} & \mathbf{u} & \mathbf{z1} & \mathbf{a} & \mathbf{b} \\ \mathbf{x} & \mathbf{v} & \mathbf{z2} & \mathbf{a} & \mathbf{b} \\ \mathbf{y} & \mathbf{u} & \mathbf{z3} & \mathbf{a} & \mathbf{b} \\ \mathbf{y} & \mathbf{v} & \mathbf{z4} & \mathbf{a} & \mathbf{b}\end{bmatrix}$$

Because of the way distributors take their effect on the context sequentially from left to right, the top two stacks will always differ in exactly one column (modulo the indeterminacy introduced by the existential). This column will correspond to the variation associated with the narrowest scoping distributor. In fact, for every distributive quantifier, there will be a stack differing from the topmost stack in exactly one position, the position controlled by that distributor's index (again, ignoring the indeterminacy of indefinites). With a little thought, it is not difficult to see that in order to compare the dual variables introduced by a certain distributor, all that is needed is to count how many other distributors intervene between the one of interest and the existential it distributes over. Every distributive quantifier introduces a single point of variation between the top two stacks, located at the column given by its index. But as other distributors are encountered, these two stacks get separated; the top stack stays where it is, but the second stack is pushed down the stacklist. And because this separation is the result of successive doublings, the relevant stack for comparison against the top stack will be the one 2^n stacks down, where n is the number of interveners.

This means that for a sentence like (9a), if we want to distribute *different* narrowly, over the girls, we direct it toward stack 1 (= 2^0, since there are 0 interveners between *different* and *every*^{1} *girl*). To distribute widely, over the boys, we direct *different* toward stack 2 (= 2^1, since there is 1 intervener between it and *every*^{0} *boy*). Whence the two LFs in (15).

Of course, given that association-with-distributivity requires passing along as much information as is required for use by adjectives like *different*, the trade-off for handling this kind of ambiguity is that the number of stacks in the stacklist must be exponential in the number of distributive operators in the sentence. But fortunately, as in Brasoveanu's fragment, all of this additional information is wiped out at the boundary of the distributor's nuclear scope.

For comparison, Barker 2007 gives a fully compositional treatment of adjectives like *same* or *different* that does not use the association-with-distributivity strategy. There is no dual quantification, and no duplication of context information. Instead, Barker proposes that the adjectives in question take 'parasitic scope':

(16) a. Every boy read a different poem.
b. (every boy) λx (x read a different poem)
c. (every boy) (different $\lambda f \lambda x$ (x read a f poem))

First, the distributive DP *every boy* takes scope, as in (16b). Then *different* takes scope in between *every boy* and its nuclear scope, as shown in (16c). Then the ambiguity in (17a) is simply a matter of how far *different* raises to find its host DP:

(17) a. Every boy gave every girl a different poem.
b. (every boy) λx ((every girl) λy (x gave y a different poem)).
c. (every boy) λx ((every girl) (different $\lambda f \lambda y$ (x gave y a f poem))).
d. (every boy) (different $\lambda f \lambda x$ ((every girl) λy (x gave y a f poem))).

Given a particular scoping of (17a), say, linear scope as in (17b), *different* is free to scope just under the narrower-scope quantifier as in (17c), taking *every girl* as its antecedent; or just under the wider-scope quantifier as in (17d), taking *every boy* as its antecedent.

3. A single lexical entry?

We can now use the differences between the fragment in Brasoveanu 2011 and the analysis here to gain a deeper understanding of how association-with-distributivity works in general, independently of any specific implementation.

Brasoveanu shows that allowing distributivity operators to spread information across context elements can lighten the compositional burden on adjectives like *same* and *different*. This enables the Brasoveanu 2011 fragment to provide a unified lexical entry for *different* which can arguably capture the similarity in meaning between internal and external readings.

(18) a. Each boy read a different book.
b. Each boy read a different book than all of the other boys read. (internal)
c. Each boy read a book different from that book. (external)

The sentence in (18a) is ambiguous between the paraphrases in (18b) and (18c). Brasoveanu suggests that one advantage of providing a single lexical entry is that it explains why languages that allow internal readings generally also allow the same lexical item to participate in external readings.

We can arrive at a unified lexical entry here as well, if we refine the lexical entry for *different* by relativizing it to a pair of integer coordinates:

(19) internal-different^n: $\lambda Pm.Pm; (\lambda c\kappa.(c_0)_m \neq (c_n)_m \wedge \kappa c)$ (same as (14g) above)

(20) external-different^l: $\lambda Pm.Pm; \underline{Pl}; (\lambda c\kappa.(c_0)_m \neq (c_0)_l \wedge \kappa c)$

(21) unified-different^{n,l}: $\lambda Pm.Pm; \underline{Pl}; (\lambda c\kappa.(c_0)_m \neq (c_n)_l \wedge \kappa c)$, where $n = 0$ or $l = m$

In the unified entry, the first integer, n, says which stack in the stacklist to find the comparison object in, and the second integer, l, says which column in the selected stack to find the comparison object in. In order to get the internal reading described above, choose

$l = m$. In order to get the external reading, choose $l \neq m$ and $n = 0$. Also, as Brasoveanu notes, we must add a presupposition (distinguished here from ordinary truth conditions by underlining) to the non-internal reading that guarantees the comparison object is a member of the category given by the complement of *different*. That is, if something is a *different poem*, then the thing it is different from must also be a poem.[5]

Is this really a unified entry? It would be more fair to say that the entry for the internal use bears a close resemblance to the external use, but they are in fact slightly different. The details in the implementation in Brasoveanu 2011 are somewhat different, involving a concatenation operator that is not necessary here, but these remarks apply equally to the implementation there.

Perhaps, though, this close but imperfect degree of similarity is the right result. It certainly makes it natural for an internal use to be generalized to an external use, without making it inevitable. Just as there are adjectives of comparison that have an external use but not internal use, as Brasoveanu notes (such as *other*, as in *John and Bill read the other book*, which cannot receive an internal reading), there may be adjectives that can receive internal readings but not external readings, such as *mutually incompatible* or *pairwise disjoint*.

Even in Brasoveanu 2011, a unified lexical entry for *same* is not possible. Unlike singular *different*, *same* can take a non-distributive plural as antecedent:

(22) The boys read the same poem.

Brasoveanu 2011:157 handles such cases with a variant lexical entry for *same* that allows *same* to provide its own distributive operator. But such a lexical item does not have an external reading. Thus even for Brasoveanu 2011, it must be possible for there to be adjectives of comparison that have internal readings but no external reading.

Finally, in addition, every account will need a separate, non-anaphoric lexical entry for both *same* and *different* to handle uses in which they appear with an overt relative clause complement, as in *a different book from the one I read yesterday*.

We should be satisfied with an analysis that emphasizes the similarities between internal and external uses of these adjectives, without trying to make one use a subtype of the other.

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[5] Note that calling a reading 'external' is inaccurate if external means that the comparison referent comes from outside of the clause containing *different*.

(i) Every^{0} boy read a^{1} book that his teacher assigned to him as well as a different^{0,1} book that he chose himself.

Here, *different* is in the scope of a distributive operator, and the identity of the book chosen by the teacher might differ for each boy. Yet *different* can select that book, guaranteeing that each boy read two distinct books (but not guaranteeing that different boys read different books).

4. Guaranteeing parallel properties of the stacks in the stacklist

In the implementation in Brasoveanu 2011, ordinary predicates like *entered* consider only the first element in the context, and ignore the remaining information. Here is how Brasoveanu puts it (p. 126):

> Furthermore, this additional information [the secondary context] is usually not accessed, even when it is available in the scope of distributive quantification. Pretty much all the updates, including the ones contributed by indefinites, pronouns, lexical relations, etc., target the left member of any input pair of info states. With one exception: items like *different* that can have sentence-internal readings.

Despite the impression given by this (accurate) description of the Brasoveanu 2011 fragment, it is crucially important to guarantee that the all the same properties and relations that hold in the primary context also hold in the secondary context.

(23) Every boy recited a different poem.

That is, in (23), it is necessary to make sure that all of the objects that will eventually be compared are poems, and that they stand in the appropriate recitation relations to the appropriate boys, both in the primary context and in the secondary context. In the implementation in Brasoveanu 2011:126, this is accomplished by the definition of the **dist** operator: in the final clause of the definition, we have that $\mathbf{dist}_{u_0}(D)\langle I, K\rangle\langle J, K'\rangle$ holds given an update function D only if for all $x \neq x', D\langle I_{u_0=x}, J_{u_0=x'}\rangle\langle J_{u_0=x}, J_{u_0=x'}\rangle$. But since $x \neq x'$ entails that $x' \neq x$, we must also have $D\langle I_{u_0=x'}, J_{u_0=x}\rangle\langle J_{u_0=x'}, J_{u_0=x}\rangle$. (Here, an update D relates an input pairing of a primary context with a secondary context with an output pairing of an updated primary context along with an updated secondary context. See Brasoveanu 2011 for the details of the '$I_{u_0=x}$' notation, which restricts an information state to those stacks that have x in the 0th column.) Focusing on the primary (i.e., the leftmost) elements of these input and output pairs, the only way that $I_{u_0=x'}$ and $J_{u_0=x'}$ can be related is if $J_{u_0=x'}$ has boys and poems in the relevant columns such that the boy in question recited the poem. This is what guarantees that the secondary context satisfies all of the properties and relations that the primary context does.

In the implementation here, this parallelism in the non-primary context stacks is accomplished in a less elegant, but more transparent way, by simply requiring each predicate to impose its requirements on all context stacks equally.

One advantage of the more straightforward technique is that it is easier to compute the update effect for concrete examples. In the Brasoveanu 2011 fragment, the obvious computational strategy is to generate all possible pairs of pairs of information states of the appropriate stack length, and then check which pairs satisfy the requirements imposed by the content of the distributive predicate; but even for toy models, this quickly becomes computationally intractable. In the fragment here, computing the set of output stacks for each input stack is straightforward and deterministic.

5. Conclusions

Association with distributivity is a viable, fully compositional account of the truth conditions of adjectives of comparison. We have discussed in detail here only singular *different*, but Brasoveanu 2011 shows how to extend the analysis to at least pural *different* and a variety of uses of *same*.

We develop a fragment building on de Groote's 2007 continuation-based dynamic semantics. In addition to providing a second implementation of AWD for comparison with the one in Brasoveanu 2011, the application to AWD illustrates the elegance, flexibility, and utility of de Groote's technique.

On the AWD strategy, the presence of distributive operators requires at least doubling the amount of discourse information tracked by the compositional semantics. On the one hand, the formal analysis here shows that it is not necessary to track full information states (sets of stacks), as in Brasoveanu 2011, since tracking simple stacks will suffice. However, on the other hand, we have argued that in the general case, the number of stacks needed must be exponential in the number of nested distributive operators. This result depends on cases in which *different* needs to take a non-local distributive operator as its antecedent.

One of the main goals of the discussion in Brasoveanu 2011 is to arrive at a unified account of internal and external uses of at least singular *different*. We have argued that the AWD strategy does not lead to a fully unified lexical entry, though it does provide a satisfying and appropriate account of the similarities across the two kinds of uses.

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